Mathematical Modeling and Culturally Relevant Pedagogy

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Abstract

Mathematical modeling and culturally relevant pedagogy (CRP) are both pedagogical approaches that rely on students' knowledge of everyday situations, yet mathematics education research has not fully attended to the ways in which they can be united in the classroom. We use an interpretation of culture as students' lived experiences, a perspective drawn from the Funds of Knowledge approach, that can uncover knowledge that is relevant for rich mathematical tasks and that can support socially-conscious reflection. This chapter proposes a new pedagogical model, suggesting that the cycle of mathematical modeling provides key moments to access students' culturally-based knowledge, and that this approach can address weaknesses in typical implementations of culturally-relevant pedagogy. Mathematical modeling asks students to complete a problem-solving cycle involving sense-making, developing problem-solving tools, interpretation and validation of results, and further cycles of model improvement. The early stage of sense-making and the reflective stages at the end of the first modeling cycle are key points at which teachers can plan discussions to foreground students' cultural knowledge and critical consciousness. We provide examples of this approach through a task on modeling neighborhood fence designs, and we provide reflections on implementing this approach with pre-service secondary teachers in an early stage of their pedagogical education.

Introduction

"The encounter with persons, one by one, rather than categories and generalities, is still the best way to cross lines of strangeness" (Bateson 2000, p. 81).

In this chapter, we propose a new pedagogical approach that brings together two domains that rely on students' knowledge of everyday situations, mathematical modeling and culturally relevant pedagogy (CRP). Culturally relevant teaching (Gay, 2000; Ladson-Billings, 1995) utilizes the students' backgrounds, knowledge, and experiences to inform the teacher's lessons and methodology, which requires teachers to create bridges between students' home cultures and the school. Through knowledge of family practices, teachers have the opportunity to connect the curriculum in mathematics and adapt various ways to learn about the everyday, lived experiences of students and their families.

Mathematical modeling is a process in which students use their knowledge of an everyday situation to engage in cycles of mathematical inquiry. Students' cultural backgrounds can play a central role within rich mathematical modeling activities, which ask students to create problem-solving methods for non-routine tasks in everyday contexts. These opportunities have the potential for teachers to leverage diverse students' everyday lived experiences for meaningful engagement with challenging mathematics through modeling tasks.

In this chapter we provide ideas that mathematics educators and teachers can use to consider contexts that are relevant to students' lives for creating mathematical modeling tasks. We initially discuss culture from an anthropological perspective and its influence on mathematics teaching and learning, followed by the tenets of CRP, with a focus on Funds of Knowledge (Greenberg, 1989; Moll, Amanti, Neff, & Gonzalez, 1992) as the approach we used in the project we describe. In this project, we engaged a group of secondary pre-service teachers (PTs) in a mathematical modeling module that brings together the tenets of CRP, culture, and local community contexts. The experiences gained by the PTs throughout the module provide a glimpse into the possibilities that teacher preparation programs can offer in the context of CRP in mathematics classrooms. We conclude the chapter with implications for teaching focusing on balancing the rigor of the mathematics, cultural connections, and helping students develop a critical analysis of the social implications.

Culture and its Influence on Mathematics Education

One of the earliest definitions of culture captures commonplace understandings of culture today, that culture is the "knowledge, belief, art, morals, law, custom, and any other capabilities and habits acquired by man as a member of society" (Tylor 1920, 1871, p. 1). In this view, culture involves the relatively consistent, visible, unchanging aspects of life--beliefs and belongings— that serve as markers of social difference among people. While this way of thinking of culture is embedded in everyday life, it proved to be insufficient for researchers and educators whose work responds to the complexities of culture.

Changing Concepts of Culture

González (2008) traces over a hundred years of the theoretical twists and turns of the anthropological culture concept subsequent to Tylor's definition. In Tylor's period, anthropologists believed that cultures evolved and improved through specific stages. The development of direct observation through fieldwork reduced this scientific racism but also strengthened the position that cultures determine human behavior. By the 1970s and 1980s, two general approaches to culture were prominent: culture as symbols and culture as activity (Sewell 1999; Henze & Hauser 1999). The first of these positions holds that culture refers to knowledge—a system for making meaning of the world. A second position focuses on people's means of taking action, "Culture is not a coherent system of symbols and meanings but a diverse collection of 'tools' that, as the metaphor indicates, are to be understood as a means for the performance of action" (Sewell 1999, p. 46).

Viewing culture as action, and as interaction, however, has not reduced the complexity of the concept. Many communities are culturally varied and foster multiple identifications; even when an individual identifies with a particular cultural group, the person may not know about or practice the elements associated with this culture (Henze & Hauser 1999). González (2008), for example, comments that "an Irish Catholic teacher can see that the Haitian family that lives next door differs in some crucial ways from a Haitian family that lives across town…the Haitian family that lives across town may be in some respects more like her own family than the Irish Catholic family that lives across the street" (p. 96). Ultimately, no authoritative framework for understanding cultural change, variation and identification has emerged in the discipline of anthropology. As anthropologist James Clifford put it, "culture is a deeply compromised concept that I cannot yet do without" (1988, p. 10 in Sewell 1999, p. 38). Researchers involved in studies of culture must define an interest within one of many dimensions of complexity.

Culture in Mathematics Education

During the 1980s, educational researchers began to incorporate social perspectives, shifting from a psychological or cognitive model of knowledge to the idea that thinking and learning are grounded in social interaction (Lerman 2000), and this social interaction serves as mediation for cognitive development from socially-guided learning (Vygotsky 1978). This "social turn" (Lerman 2000) was grounded in a concern with acknowledging and addressing social inequality in research and in classrooms. Despite the intractability of the definition of culture, many educators regard the culture concept as vitally important for improving equity in schooling. In some respects, this shift recalls the debate of culture as knowledge versus action. Mathematical knowledge was viewed as the product of action, discussion and construction, rather than simply as an intergenerational transfer of knowledge.

Bishop (1988), for example, argues that mathematics is a cultural practice. He suggests that several types of cultural activities can lead to culturally-based mathematical ideas: counting, locating, measuring, designing, playing, and explaining. He proposes that a "culturally-fair" curriculum could be designed from the standpoint of this structure, which would allow local mathematical concepts to enter the classroom, along with widely-shared forms of academic mathematics. "Is it indeed possible by this means to create a culturally-fair mathematics curriculum—a curriculum that would allow all cultural groups to involve their own mathematical ideas whilst also permitting the 'international' mathematical ideas to be developed?" (Bishop 1988, p. 189). The field of ethnomathematics, too, addressed issues of cultural fairness through ethnographic inquiries into mathematical practices embedded within cultural activities (Ascher 1991; d'Ambrosio 1985, 2006). Ethnomathematics faces the conundrum that activities are most clearly recognized as mathematics when they are translated into traditional mathematical forms (Civil 2014; Wagner & Lunney Borden 2012). Though this issue is unresolved, several scholars have recommended the general approach of asking community members to identify activities that they consider mathematical, and to develop curriculum from this starting point (Borba 1997; Wagner & Lunney Borden 2012).

As culture is embedded in issues of fairness and equity, the unruly nature of the concept creates tensions in basic questions such as what activities or representations of activities count as mathematics, and how educators can incorporate mathematical community knowledge into classrooms as a bridge to widely-recognized mathematical practices.

Culturally Relevant Pedagogy

Culturally Relevant Pedagogy has become one of the most influential responses to incorporation of cultural perspectives in education. By structuring curriculum and classroom interactions around students' cultures, CRP seeks to ensure that students are academically successful and that they develop a sense of social critique (Ladson-Billings 1995). CRP emphasizes the development of a collective rather than individualized identity (Ladson-Billings 1995; Tate 1995). The idea is that through a "pedagogy of opposition" (Tate 1995, p. 169), students resist assimilation into the cultural norms of the majority and use classroom learning to take action in their communities.

CRP calls for developing pedagogical approaches in which students: (a) experience academic success; (b) develop and/or maintain cultural competence; and (c) develop a critical consciousness through which they challenge the status quo of the current social order. These three tenets constitute the basis for using students' strengths to promote academic success. Several researchers have used CRP in mathematics education, including Greer, Mukhopadhyay, Powell, and Nelson-Barber (2009), Gutstein, Lipman, Hernandez and de los Reyes (1997), Lipka, Yanez, Andrew-Ihrke, and Adam (2009), Moses and Cobb (2001), Tate (1995), and Turner and Font Strawhun (2007).

Dilemmas Posed by Culturally Relevant Pedagogy

Although CRP has become one of education's "best practices," its complexity means that it is often implemented in ways that diverge from its original principles. Of Ladson-Billings' three goals, cultural competence has been attended to more strongly than the other two principles (Young 2010). However, a limited perspective on culture, similar to Tylor's 1871 definition, underlies some of the problem. CRP misses the point when it merely involves "acknowledging ethnic holidays, including popular culture in the curriculum, or adopting colloquial speech" (Irvine 2010, p. 58). This can have the effect of emphasizing "the sense of otherness commonly felt by minority students" (Young 2010, p. 252; referring to Troyna 1987). Further, teaching practices for CRP are often developed in reference to homogeneous classrooms (Morrison, Robinson & Rose 2008). Teachers may assume that all students identify with one version of one culture. More broadly, a focus on cultural difference may reproduce a system of exclusion if teachers assume that different children require different pedagogies, or if the target of academic development is to achieve a standard defined as the behaviors and level of achievement of the dominant group of students (Schmeichel 2012).

Aguirre and Zavala (2013) have addressed this dilemma through a lesson analysis tool that uses all three tenets of CRP to help teachers integrate mathematical thinking with components of CRP such as language, culture and social justice. These authors refer to Culturally Responsive Mathematics Teaching (CRMT) as "a set of specific pedagogical knowledge, dispositions, and practices that privilege mathematical thinking, cultural and linguistic funds of knowledge, and issues of power and social justice in mathematics education (Aguirre & Zavala 2013, p. 1)." It remains a challenge to reach consensus on the content of such CRMT tools and how to prepare teachers for integrating CRP principles into mathematics instruction.

In general, critical consciousness is the component of Ladson-Billings' model that is less-fully realized in classroom teaching (Young 2010). Teachers may feel uncomfortable with political analysis—many people in the United States prefer discussing culture instead of structural inequity or racism (Sleeter 2011). Reflection on personal identity is a necessary step for teachers from dominant social classes before they can implement classroom activities that support development of critical consciousness among diverse students.

Addressing Dilemmas through Funds of Knowledge

In recent years, educational researchers have begun to acknowledge the difficulty of implementing each of the three elements of CRP. Nuanced understandings of culture, teaching across cultural differences, and integrating cultural and mathematical understanding are significant dilemmas for this pedagogical approach; debates over the meaning of culture are at the heart of all of these issues.

The Funds of Knowledge approach (González, Moll, & Amanti, 2005; Greenberg, 1989; Moll, Amanti, Neff, & Gonzalez, 1992; Tapia, 1991) can address some of the difficulties in implementing the three tenets of CRP. Through ethnographic visits to some of their students' homes, teachers learn about their students' and their families' knowledge and experience their funds of knowledge. This process places families as the knowledge experts and the teacher as a learner.

Following the Funds of Knowledge approach, we adopt González' dynamic view of culture as "lived experience. The focus is on 'practice,' that is, what it is that people do and what they say about what they do. The processes of everyday life, in the forms of daily activities, emerge as important" (2008, p. 96). This perspective asks teachers, researchers and students to actively investigate the forms that culture takes in a particular community.

The Funds of Knowledge approach overturns deficit concepts of students. By identifying reservoirs of community expertise, and creating projects and classroom activities around them, teachers can engage students' knowledge more deeply. Because it involves teachers' active learning in households and communities, this approach may be able to uncover cultural complexity of communities better. It avoids the pitfalls of homogeneity, and it accounts for cultural change and cultural borrowing. It can help teachers, develop critical consciousness within themselves and prepare them to help students find their political voice. The active search for community knowledge represented in the Funds of Knowledge approach can avoid essentializing assumptions about students' cultures.

Examples of mathematics education work within the Funds of Knowledge approach include the use of occupational interviews to uncover the mathematics behind some practices (e.g., a mechanic, a carpenter, a seamstress) (Civil & Andrade, 2002; Civil, 2016; González, Andrade, Civil & Moll, 2001). Civil (2007) describes two mathematically-rich classroom experiences based on funds of knowledge work, one centered on construction with a class of second graders (see also Sandoval-Taylor, 2005), that involved geometric thinking and measurement; the second one was a garden unit with a class of fourth and fifth graders, also exploring ideas of measurement and optimization (maximizing area of a garden plot given a fixed perimeter) (see also Civil & Kahn, 2001). This chapter draws on the general concepts at the heart of Funds of Knowledge to propose an approach that brings together mathematical modeling and CRP. We next turn to a discussion on mathematical modeling.

Mathematical Modeling

Mathematical Modeling: Its History and Background

The mathematics literature has long discussed ancient cultures that used modeling to improve their everyday life starting around 2,000 B.C. (Schichl 2004). In this context, modeling meant the application of mathematics to solve problems arising in sciences (e.g. astronomy) and other aspects of everyday life. For centuries, mathematical modeling has been driven by the desire to describe nature's principles. More recently, the motivation for developing mathematical models comes from an increasing number of disciplines including the sciences, technology, engineering, economics, health care, politics, and more. Today, modeling is an area of mathematical research and it is typically taught in universities as part of an applied mathematics curriculum.

By the mid-1980s, mathematical modeling was emerging in the U.K. and Europe as a pedagogical approach in secondary and early undergraduate mathematics curriculum (Berry, Burghes, Huntley, James, & Moscardini 1984). The approach emphasized an active and creative way of learning mathematics—"learning modeling"—rather than memorizing established approaches to solving formulaic problems—"learning models" (Burkhardt 1984). In the USA the National Council of Teachers of Mathematics (NCTM) underscores the use of representations to interpret physical, social, and mathematical phenomena in mathematical modeling (2000).

Perspectives and Definition of Mathematical Modeling

Drawing from Lesh and Zawojewski (2007), English and Sriraman (2010) write that "modeling problems are realistically complex situations where the problem solver engages in mathematical thinking beyond the usual school experience and where the products to be generated often include complex artifacts or conceptual tools that are needed for some purpose, or to accomplish some goal" (p. 273). In the Common Core State Standards, model with mathematics is one of the Standards for Mathematical Practice and is defined as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (CCSSI 2010, p. 72). The expectation is that all elementary and secondary school students will develop modeling proficiency, which includes applying the mathematics they know to solve problems not originally posed as mathematics problems, and making simplifications and choices that must be validated and possibly revised. Several authors have written about modeling implications and issues connected to the Common Core (e.g., Anhalt & Cortez 2015; Felton, Anhalt, & Cortez 2015; Tam 2011). In the Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) report (Garfunkel & Montgomery, 2016), define mathematical modeling as a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

It is important to recognize that mathematical modeling is not only a part of K-16 education but also an active area of research among professional mathematicians. For this reason there are multiple sources that define mathematical modeling. The definitions have some variations but are essentially very similar: mathematical modeling is an iterative process whereby we use mathematics to understand or analyze some situation that often comes from outside mathematics. To illustrate the definition, consider the following situation:

The weather forecast calls for heavy rain for several hours. It is expected that the water level of a river that goes through town will rise above its banks in one particular section and cause major flooding, so the residents want to protect themselves from the flood by elevating the riverbank using sandbags. How long will this take?

To estimate the answer, we can use mathematics. If we can find out the dimensions of the bags filled with sand, the proper way to stack them, the desired height of the sandbag wall, and the length of the river section that needs to be protected, we could develop a mathematical formula that tells us how many sandbags we might need. We can then estimate the time it will take to fill the bags and build the protection wall depending on the number of helpers and additional assumptions. The formulas themselves constitute the model in this case. The entire process is mathematical modeling. Even after doing this, we may find that additional variables or parameters need to be taken into account and the formulas will have to be adjusted. For example, the sandbag thickness near the bottom of the wall may be smaller due to the weight of the sandbags on top; or the time between filling the bags and placing them on the wall may get longer as the wall grows. Such adjustments to the model can be made iteratively.

Elements of Mathematical Modeling

One of the components of the modeling process is the formulation of a model. A mathematical model is a "simplification of reality that is phrased in the symbolic language of mathematics [that] can take the form of equations, algorithms, graphical relations, and sometimes even paragraphs" (SIAM 2012, p. 11). In education, a definition of *models* is given by Doerr and English (2003) as "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (p. 112). The modeling process, however, has additional elements that are usually represented as stages of a cycle like those shown in Figure 1. Similar representations have been depicted in many sources, including textbooks (Mooney & Swift 1999) and mathematics and mathematics education journals (e.g. Blum & Leiss 2005; CCSSI 2010; Meier 2009; Yoon, Dreyfus & Thomas 2010; Felton et al. 2015).

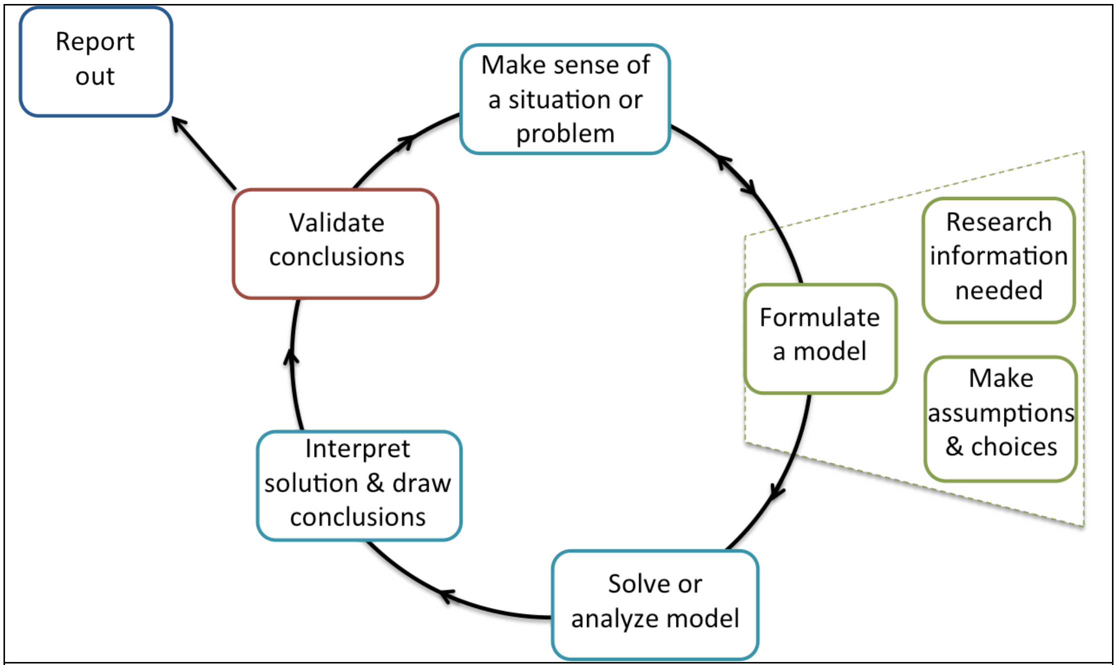

Figure 1. A representation of the mathematical modeling process.

The figure emphasizes the iterative nature of the modeling process and identifies salient elements, all of which could be expanded further. Starting with a situation to be analyzed, the first stage is to make sense of it and understand the questions that need answers. The next major stage is the formulation of a model, which involves formulating a mathematical problem that represents a simplified or distilled version of the original situation. The model formulation step may involve sub-stages such as determining essential variables, making assumptions about any missing information, and choosing appropriate mathematics (e.g. statistics, linear functions, etc.) for the model. Typically, sense-making continues during this part of the process and some research is necessary to make reasonable assumptions.

Once the model is formulated as a set of equations, a graph or a table of values, the problem-solving step leads to a mathematical solution that needs to be interpreted in the original context. Conclusions about the original situation are drawn from this interpretation and the conclusions must be evaluated in a validation stage in order to determine if they make sense in terms of the original situation. Since the mathematical answer is influenced by the assumptions and choices made earlier, the conclusions may not be satisfactory based on the needed accuracy, the applicability of the solution or some other factor. If this is the case, a new iteration is entered where assumptions and choices are revised with an eye on overcoming the shortcomings of the first model. The cycle may be repeated once or multiple times until satisfactory conclusions are reached and can be reported.

Mathematical Modeling and Culturally Relevant Pedagogy

Culturally relevant pedagogy is based on the assumption that when academic knowledge and skills are situated within the lived experiences and frames of reference of students from various cultural backgrounds, they are more personally meaningful, have higher interest appeal, and are learned more easily and thoroughly (Gay 2000; Ladson-Billing 1995). Although CRP and mathematical modeling are both significant and well-respected contemporary pedagogies, there has been relatively little explicit scholarship on ways to integrate their strengths. The cycle of mathematical modeling is inherently challenging, reflective, and it depends on contextual knowledge of everyday situations. For these reasons, we suggest that mathematical modeling corresponds naturally to the tenets of CRP. In particular, mathematical modeling pedagogy can address some of the weakness of implementation of CRP that has been observed of the past few decades. Mathematical modeling activities: (a) motivate mathematics content, (b) promote discussion between students, and (c) integrate contexts relevant to students. In what follows we describe how these three characteristics of modeling relate to CRP.

Motivating mathematical content**.** A modeling task can address specific content and build upon content previously learned. At the same time, because the models developed by students are limited by their mathematical knowledge and experience in recognizing essential variables and their relationships, the task also serves as a springboard for discussing content that is new to students. By grounding models in students' lived experiences, the cultural context can motivate students—to offer them a reason to conduct mathematical activities. Ladson-Billings' intention was that CRP would be implemented holistically, and that each of the three tenets would support each other. Conceptualizing modeling in this way helps strengthen this dimension of CRP, that students must succeed in a rigorous academic environment, and that personal knowledge is a factor that leads to this success.

Promoting discussion between students. The modeling process promotes mathematical discourse as it requires justifying the choices and assumptions made along the way, the selection of variables and mathematical concepts for the model, and the choices of representations. Substantial communication is also needed to report the solution and critique others'. Appropriately designed modeling problems provide opportunities for students to actively use mathematical language to communicate meaning about and negotiate meaning for mathematical situations. Culturally-based modeling activities touch on students' cultures and demand that students communicate the connections between the context and the mathematics they have used in their models. This promotes understanding of students' cultures and brings the significance of students' cultural background to the foreground. Teachers who intentionally plan modeling discussions can include topics that assist students' integration of cultural knowledge as it is realized in community and household activities.

Integrating relevant contexts. The entire modeling activity may be motivated by activities that are familiar to students. That is, modeling allows us to draw on students' funds of knowledge and design activities that are culturally relevant. The modeling process contains opportune moments to draw cultural knowledge into mathematical problem solving. In the initial stages of the cycle, students usually establish simplifying assumptions that allow them to create their mathematical models. Some of these assumptions are motivated by mathematical needs—students will choose to use mathematical structures that they understand—but also, they will be based on students' knowledge of the context of the task. Explaining why an assumption is reasonable will rely partly on students' lived experiences.

To complete the first cycle of the modeling process, students reflect on the strengths and weaknesses of their model, they discuss whether their solutions are reasonable, and they plan a subsequent cycle of improvements to the model. These interpretation and validation stages of modeling are moments in the modeling cycle in which students' contextual knowledge is important. Typically, validation involves critical reflection on the mathematical scope and accuracy of the model but this critical reflection could extend to questions of equity, access, and fairness when these concepts are relevant to the model's context. We suggest that the critical reflection of the final stages of the first modeling cycle are appropriate times for the teacher to engage students in discussions to explore and strengthen critical consciousness.

When exploring ideas taken from students' background cultural knowledge, a natural, yet sometimes uncomfortable next step is to ask students how they view the idea in the context of the world views, such as taking into consideration the political, social, and/or economic perspectives that are associated with the problem situation. Exploring these aspects of the problem situation is important, yet it is essential to focus on how the mathematics helps explain the situation. Integrating CRP into the modeling cycle can show students how to use mathematics to interact more powerfully in the social world, so that mathematical modeling activities can promote taking action.

Strengthening Implementation of CRP through Mathematical Modeling

CRP is a significant advancement for connecting lived experience to mathematical explorations, but as we have seen, there are challenges in the implementation of each of its three tenets. Teachers must be able to implement all three tenets in an integrated way so that they inform one another. Incorporating a strategy for culturally-relevant teaching into the mathematical modeling cycle can address some of these dilemmas. Modeling improves the rigor of the curriculum, and the modeling cycle allows teachers to plan discussions that address cultural competence and critical consciousness at specific stages. Using this approach, teachers can attempt to implement Ladson-Billings' construct in the manner in which it was intended. In the following section, we describe a mathematical modeling activity involving mathematical functions in a community cultural context, along with topics that have the potential to raise critical questions about the social world.

Community Contexts for Mathematical Modeling

This modeling activity was created as an illustration of a problem that allows students to experience the mathematical modeling process as they work through the problem.

Neighborhood fences and gates: Design using mathematics

As you walk around your neighborhood, you will see lots of fences and gates in the front yards of houses. The design of the fences can be described by mathematical functions.

1) Walk around your neighborhood and take pictures or draw sketches of yard fences or gates of different shapes. If your house has a fence be sure to include it.
2) Find mathematical functions that can be used to design the fences in the pictures or sketches. Include the domain of the functions. Since the pictures don't have coordinate axes, you will need to make choices about the height and width of your functions and possibly about other parameters. Be sure to list the choices you make and your reasons for making those choices.
3) Fences and gates can have different purposes. Use your imagination to sketch or describe a new fence shape that you find interesting and that has a unique shape. Think of where your fence might be used and what purpose your fence might have. Then find a function that describes your fence and explain the choices you made and how those choices are connected to the purpose for your fence.

4) Provide a set of instructions that you can give to a picket fence builder. Your instructions should include the number of pickets and their width, the height of each picket, the separation between pickets and the order in which they should be installed.
5) Look for public places or private homes in your neighborhood that do not have fences. Propose a fence or a gate for one of these places based on its purpose and provide a mathematical function for it.

Figure 1. The “neighborhood fences and gates” problem.

There are several reasons why this problem is a good choice as a modeling activity. First, there is no particular correct answer that students must find. In fact, the problem does not ask to find a number or a specific expression or formula. Instead, the students are asked to propose functions that have qualitative features, which allows students to use creativity and prior knowledge to suggest functions. Since there are multiple correct possibilities, there is opportunity for students to reveal their knowledge and personal choices.

Second, the mathematical modeling requirements are the same regardless of the specific pictures of fences that the students bring. However, the variety of options that can result from different fence shapes and different gate purposes provides multiple directions for students to explore and opportunities to justify specific choices to formulate their models. It is known that “different purposes may result in different mathematical models of the “same” reality (Jablonka, 2007, p. 193).” For instance, a decorative fence may be relatively low and have more spacing between pickets compared to a security fence or a fence to keep a pet from running out of the yard. These considerations affect the choice of parameters needed for the model. As an example, Figure 2 shows photographs of fences and a gate with different heights and shapes. Based on observations of the photos, some students may choose to create a single function for the entire fence while other students may reason that a fence is made of repeating segments and choose to provide a function for the segment only. Additionally, the parabolic-looking fence on the right photograph can be modeled by a polynomial, a trigonometric function or some other curve. These choices are part of model assumptions.

Figure 2. Examples of neighborhood fences and gates.

Third, the functions that students produce constitute an initial model. The interpretation of the graph of their functions as the shapes of fences or gates can conclude a first pass of the iterative modeling process. There are several options for revising the model, some based on the shapes (*are the functions high enough? Do they dip too low in some places? Are they aesthetically pleasing or should they be modified?*) and some based on the representation of the functions. For instance, a bar graph of a function may give a better visual idea of what the fence will look like. Figure 3 shows two representations of the same function for values of x in the interval [-1,1]. The bar graph on the right gives a better sense of the fence. Refinement in the functions themselves or their representations can be attempted as iterations within the modeling process.

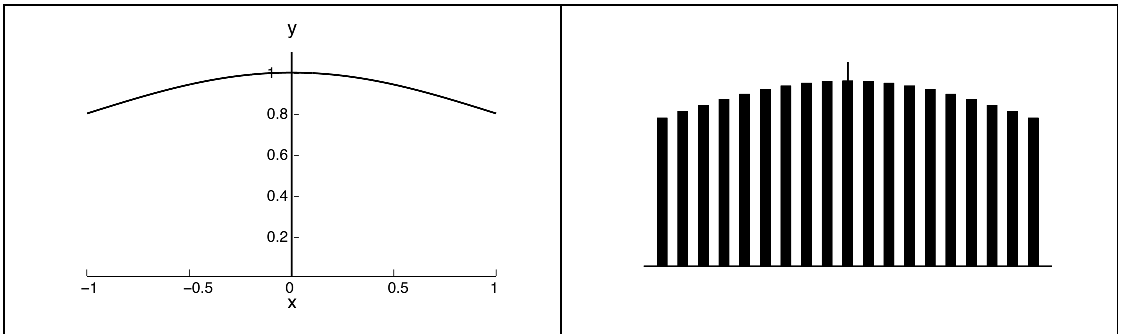

Figure 3. Example of two ways of representing the function y(x) = $\frac{1}{5}cos(\frac{\pi x}{2}) + \frac{1}{4}$.

Mathematical Content of the Task

The Fences task involves substantial content related to functions, some of which is mentioned explicitly and some that is intended to surface as the students work on the task. The domain of the function is explicitly requested and it is directly connected to the mathematical modeling assumptions. For instance, if the bottom of the fence is the *x*-axis and the top of the fence is given by the graph of $y = f(x)$, the domain of the function $f(x)$ determines how wide the fence will be as well as the minimum and maximum heights of the fence. If the fence is assumed to be at least 5 feet high, the range of the function must satisfy this assumption.

A traditional question might provide a specific function $f(x)$ and ask the students to determine the domain and range. The Fences task is perhaps more challenging since it asks the students to produce a function whose range has particular features like "it cannot include values of y less than 5." The students also have to make assumptions regarding the height and width of the fence segments and translate those assumptions into the range and domain of their functions.

Part #3 of the activity is wide open for students to explore new functions and provides the opportunity to discuss concepts like even functions, functions that are neither even nor odd, periodic functions, piecewise functions, etc. Part #4 addresses the more practical side of the activity and expects students to generate a table of values for each of the fence pickets to be created.

Connection to Culture and Community

Fences are part of residential landscapes that contribute to a community's cultural space. For example, the housescape, including house colors, religious images, decorations and fence enclosures, are common features of many Latino neighborhoods (Arreola 2012). A survey of two neighborhoods in central Phoenix revealed that "Sixty-nine percent of front yards in Garfield [mostly Hispanic] were completely enclosed, with only 18 percent of those in Coronado [mostly non-Hispanic] fenced" (Manger 2000, p. 6) but residents in both neighborhoods perceived the purpose of fences to keep pets or children in the yard, to keep trespassers out, or to demarcate boundaries (Manger 2000).

The Fences task asks students to look through their neighborhoods for examples of fences or gates and to consider their purpose. In this way, the students will bring a piece of their neighborhood to the classroom and share it as part of the activity while simultaneously considering cultural implications of the purposes for fences. Throughout the problem, the focus of the activity is the mathematical functions that represent the tops of fences and the connection between mathematics to aspects of the students' lives.

Connection to Critical Consciousness

Part #3 of the problem alludes to the fact that some fences may be purely decorative or have purposes related to security or privacy. Without explicitly mentioning these purposes, the problem lets the students suggest possibilities and opens the door for a discussion on perceptions of crime in neighborhoods and social implications of such perceptions. Importantly, this part of the problem is not divorced from the mathematics as it asks students to think about and justify how the purpose of the fence/gate affects the mathematical choices they make in their design. The last part of the problem (item #5) makes a connection between the students' findings and suggestions to a concrete action that may improve or otherwise effect a change in their neighborhoods. The purpose that students cite may be aesthetics, security or something else. Throughout the process, the problem emphasizes the mathematical knowledge required of the students.

Connection to Academic Success

As aforementioned, the problem addresses functions and function representation in a nontraditional way. In contrast to traditional textbook problems which typically provide a function to be graphed or provide sufficient information to determine a unique function, this task requires students to suggest functions that have certain general features, which may be met by several functions. Students must understand how to produce functions with the given features and, further, provide new features of their choice and produce functions that meet them. This kind of task requires a high level of understanding of functions.

Implications for Teaching: Balancing the Tenets of Culturally Relevant Pedagogy

Practicing CRP in the context of mathematical modeling may seem like a daunting task to many teachers since both are demanding in terms of time and knowledge. Nevertheless, we have made a case for the natural integration of CRP and mathematical modeling because teaching mathematics with the expectation that all students succeed academically is at the heart of both. Since mathematical modeling draws on students' mathematical knowledge while offering opportunities for new mathematical content to be developed, teachers can support students in critical thinking about their approach to mathematical modeling. For this reason, modeling tasks have the potential for teachers to leverage diverse students' everyday lived experiences for meaningful engagement with challenging mathematics. The way the students maneuver around the modeling process is informed by their culture and 'ways of thinking' which are formed by their everyday lived experiences.

As students show evidence of logical reasoning, especially for improving their initial models by re-evaluating their assumptions, teachers can use this opportunity to extend student thinking and ask for justification, motivation, and explanation of the improvements. Given that any mathematical model can be improved in some way, classroom discussions can develop both critical consciousness and mathematical strategies once students have completed the first cycle of modeling. The following questions are designed to help teachers chart a discussion pathway from the mathematics that students use to self-awareness of how culture influences their decision-making to social consciousness and critical views of the world.

- What mathematics did you use to create your initial model of the problem?
- What other mathematics could you have used?
- How is your model similar or different to other models created by peers?
- What information did you need to research to make assumptions for your model?
- What influenced you to choose the assumptions you came up with?
- What "ways of thinking" from your background knowledge and culture impact your decisions in the modeling process?
- What aspects of your model do you think can be revised and improved?
- What aspects of your model help you think about social issues that impact people in various places in the world? These social issues could include issues related to economic, social equity, fairness, safety and protection, political influences in people's lives.

As a concrete example, the following are possible questions about the Fence task presented earlier as it relates to middle or high school students:

- What did you notice about the type of fences and gates that you found? From what materials are the fences made? What is the purpose for the fences that you found?
- What do you think is the cost of these different kinds of fences?
- What is the relationship between the cost of the fences and the design of the fences? What is the relationship between the cost of the fences and the purpose for the fences?
- If your family wanted to put a fence in your front or back yard, what would you choose for materials or design? How could you determine the cost of the materials?
- How much artistic or aesthetic value would you like your fence to have? Is this important to you or your family?
- When families settle in a new country, are there costs involved for people who want to maintain aspects of their culture? How could this affect fence choices?
- Which of our class fence designs would cost the most? The least?
- If you wanted to make your fence more culturally aesthetic, and only increase the cost by a little, how would you do it?

These discussion questions attempt to tie choices about cultural conservation and aesthetics to household financial decision-making. In many case studies of culturally relevant teaching, the discussion begins with students identifying problems in their community (e.g. Ladson-Billings 1995; Tate 1995; Turner & Font Strawhun 2007; Turner, Varley Gutiérrez, Simic-Muller, & Díez-Palomar 2009). In the mathematical modeling context, discussions of critical consciousness can occur between the formulation of an initial model and making decisions for possible improvements. This teaching trajectory allows the teacher to observe the type of mathematics that the students use in the initial model and then guide them to increase the level of mathematics, specifically when the teacher knows the kinds of connections that students could make to improve the model. This is a useful strategy when teachers feel pressure to align student mathematical work with curriculum standards. Conducting a critical consciousness discussion between modeling iterations also helps achieve the original intention of creating a unified sense of purpose for mathematics and critical consciousness.

Implications for Teacher Education

Achieving a balance of rigorous mathematics content, cultural competence, and critical consciousness through mathematical modeling is a complex endeavor, yet an attainable goal that needs much attention. It is necessary for teacher education to provide experiences with mathematical modeling that can prepare teachers to engage their students in the mathematical modeling process. Two critical aspects of this teacher preparation are: (1) becoming comfortable posing modeling problems that are open-ended and different from traditional textbook problems, and (2) understanding the concept of making assumptions, as this is something that they may not have experienced explicitly before in mathematics.

For effective teacher preparation, teacher educators must become fluent with the nature of the mathematical modeling cycle as an approach to solving open-ended problems in familiar contexts. In order to promote creativity, teacher educators should resist steering teachers toward predetermined modeling approaches; rather, support their own thinking to develop their models. Time should be taken to uncover how teachers' backgrounds influence their modeling approaches and to have open discussions with teachers about their cultural influences on learning, especially in decision making during mathematical modeling. For teacher professional development, it is important to include projects in which teachers of various grade levels in K-12 collaborate to experience diverse mathematical modeling tasks to develop understanding of the modeling process while simultaneously work toward inclusion of the CRP tenets.

For prospective and in-service teachers, understanding the various aspects of CRP can take place through readings, discussion, and engagement in a problem-based project to bring together the various elements. Building on these components, teachers can strategize and build a progression for ways of teaching mathematical modeling with the relevant cultural aspects that help shape critical consciousness for students. Further deepening of this aspect would require implementation, analysis of student work, continuation of collaboration through discussions, building more context-rich and relevant modeling tasks, and continuous reflection for improving the teaching of mathematical modeling.

A Brief Look at the Mathematical Modeling Module with Cultural Aspects

We close this chapter with a brief look at the module we implemented with a group of preservice teachers (PTs), in a sophomore-level mathematics pedagogy class in a department of mathematics. Because none of the PTs in the class had taken a mathematics course in mathematical modeling nor had they had course work relating mathematics and culture, we assumed that most of the ideas would be new to most of them. We first introduced the construct of culture by assigning a reading, "What is Culture?" by Gonzalez (2008), to discuss the role of culture on the learning and teaching of mathematics. This centrality of culture was followed by the introduction of mathematical modeling problems that touched on cultural aspects.

Consistent with the Funds of Knowledge approach, our definition of culture was that of lived experiences. We had several activities to engage students with discussions around culture, including having students build individual "Identity Maps" to share their individuality; readings (Gay 2002; González 2008); a video (Teaching Tolerance 2010); discussions around major points pertaining to culture and CRP; guest speakers (Norma González on culture and Funds of Knowledge; Marta Civil on topics of culture and Funds of Knowledge in the mathematics classroom). The PTs' immediate questions and concerns were about the teaching of secondary mathematics concepts and how to include culturally relevant aspects.

We followed this with a mathematical modeling problem that tied in with culturally relevant aspects, "'Cuts & Styles' is a hair salon that claims to serve over a million customers per year. Is this reasonable? Under what conditions could this be true? Create a mathematical model for this situation." This simple and open-ended problem required the PTs to consider many assumptions drawn from their knowledge and experiences in hair cutting including knowing particular owners of local salons. These experiences were shared in small group discussions before formulating a model. This problem allowed for PTs to analyze and compare traditional textbook problems with aspects of this particular problem. The assumptions were based on their personal experiences of getting haircuts and considering the variables involved, such as the location (to determine how busy the place could be) and the number of minutes for haircuts and styles for short and long hair. Economics became part of the discussion on cost of haircuts and styles; some students claimed that they did not cut their hair often because of the expense, which also led to discussions about various places and the cost associated.

Another assignment included reading and discussing the pertinent pieces of the Common Core State Standards in Mathematics (CCSSI 2010), specifically the mathematical modeling cycle as described in the high school conceptual category (pp. 72-73), and the K-12 mathematical practice, Model with mathematics (p.7). In addition, the PTs engaged in transforming traditional textbook problems into modeling problems. This proved to be challenging because there was a sense of uneasiness with leaving out parameters and posing open-ended problems. One example of a traditional textbook problem (Jacobs 1982, p. 140) that was given to the PTs was the following:

A person's shoe is a function of the length of his or her foot. Formulas for this function for men's and women's shoes are given below: x represents the length of a person's foot in inches and y represents the corresponding shoe size.

Men's shoe size is $y = 3x\text{-}25$.

Women's shoe size is $y = 3x\text{-}22$.

(a) Graph both functions on one pair of axes. What do you notice about their graphs?

(b) If a man and a woman have feet of the same length, who has the larger shoe size?

(c) If a man and a woman have the same shoe size, who has the longer foot?

Following this activity, the PTs engaged in the Fence mathematical modeling activity shown in earlier in this chapter. The PTs shared that this task solidified their understanding of the community context and its relevance to mathematics learning. Each PT found images of fences and gates around their own community (mostly around the university campus) and created functions related to the designs of the fences and gates in their images. This activity promoted much discussion on the elements of modeling, mainly around the notion of creating function models of the top edge of fences and within their designs. Additional discussion was around the purpose of these fences in their communities including the safety of their neighborhoods, and possibly how the taller and less aesthetic fences correlated to some kind of safety factor. We recognize that PTs are not traditionally asked to consider culture in preparing mathematics problems, so this example proved to be fruitful in underscoring the tenets of CRP and the elements of mathematical modeling.

In the end, it was evident that one module in one course with several mathematical modeling example problems may not provide enough experiences for PTs to feel fully comfortable or confident in incorporating the components of CRP. Several PTs indicated that they understood how the cultural backgrounds of students can have a role in the learning process of modeling problems, but that the critical consciousness connections with mathematics were less developed for them. This general reflection from the PTs lead us to believe that more mathematical modeling problems that incorporate the CRP tenets are necessary to help PTs develop their understanding of the mathematical modeling process for teaching it through the lens of academic rigor in mathematics while integrating students' cultural backgrounds, and helping students develop critical consciousness of meaningful social issues.

There are some limitations to integrating the practice of CRP with mathematical modeling. First, while modeling is prominent in mathematics applied to everyday situations, it is more difficult to connect abstract mathematical concepts to cultural knowledge and attempting to do so may over-simplify either the mathematics or the cultural understanding. The PTs who engaged in the module made this observation. Second, the modeling process requires students to translate back and forth between the situation context and a mathematical model. This translation is informed by students' lived experiences but once a mathematical model is constructed, the students enter a problem-solving realm in order to compute a solution of the equations - or other mathematical constructs - in the model. This stage can be unrelated to the context of the problem (some algorithms used to solve problems can have cultural connections). Consequently, the link between culture and mathematics is temporarily interrupted, which can cause a loss of continuity in the CRP process. Similarly, a disproportionate emphasis on the social issues that a situation evokes can relegate mathematics to a mere tool rather than a discipline whose understanding must be solidified and expanded.

Conclusion

Through rich mathematical modeling problems, students are able to work within the tenets of CRP: achieving through mathematics, building cultural self-awareness, and developing critical consciousness. Various elements of the mathematical modeling process require students to make decisions, for example, formulating a model requires that assumptions be made, or in interpreting and validating the model after analyzing the results. We argue that students' background knowledge including cultural backgrounds, lived experiences, and mathematical knowledge inform the modeling process. Because mathematical modeling requires students to consider relevant information they may know about the problem situation, decision making, formulating a model, and finally interpreting and validating the outcomes, we argue that the process requires ownership of the mathematics and navigation through the modeling cycle. Students bring in their 'ways of thinking' about the mathematics and the social contexts and implications that the problem situation presents to them.

By incorporating rich mathematical modeling problems that involve students' researching of their own communities, we can provide opportunities for learning school curriculum mathematics in a way that is most relevant to students. Having specific knowledge of the cultural background of the students in a class makes it possible for a teacher to present modeling tasks that connect to the students' lives and promote discussions about issues that students care about. Once the students take ownership of a problem, they can engage in more meaningful discussions about the mathematics and other social issues that may be important to them.

Mathematical modeling can be thought of as a way to bring together a set of mathematical concepts, selected by the students, and apply them strategically to address a situation that comes from any part of life. This freedom to use mathematical reasoning to address issues in contexts outside typical school mathematics is precisely why mathematical modeling lends itself nicely to CRP. Aspects of students' cultures related to school regulations, social inequities, truth in advertisement, hobbies, health, etc. can be investigated and discussed with the use of mathematical models. Learning to use modeling as a framework for accessing students' funds of knowledge, as Bateson (2000) would have it, helps teachers lead students across lines of strangeness into a world of socially-aware mathematical exploration.

Teachers may encounter some tension between incorporating authentic cultural knowledge into the modeling process while staying true to the goals and modes of analysis of the discipline of mathematics. Other chapters in this volume allude to this tension—or aspiration—in the contexts of science and engineering education. In chapter 4, for example, Sjöström and Eilks provide a nuanced discussion of the dimensions of critical-reflexive *Bildung* in science education, the knowledge of self, society and capacity for action. The principle of *Bildung* resonates with culturally responsive pedagogy through the valuation of increased awareness of a cultural self, and the understanding that STEM disciplines can create a better and more just world. Purzer, Moore, and Dringenberg, in Chapter 8, describe engineering design as an iterative process that alternates between acquiring and applying knowledge (Fig. 8.3). The knowledge acquisition stage recognizes that the initial problem statement will be ambiguous and partial, so that students need to learn to question and communicate deeply with the client. In chapter 10, Carberry and Baker recognize that engineers need to engage users more deeply than the discipline sometimes values, to become sensitive to cultural, economic and power-laden fault lines that can sink an engineering project. Awareness of the Funds of Knowledge approach with its direct and deep engagement in communities could contribute to culturally-sensitive design processes in many STEM fields.

While we stress that there is no consensus on modeling pedagogies in any of the STEM fields, we also note that those fields that use an iterative design or pedagogical process may be able to incorporate perspectives from our chapter. Our proposed pedagogical model asserts that the stages of mathematical modeling provide valuable moments to access students' culturally-based knowledge and to use this knowledge as a resource for learning. We offer this approach as a step forward in the development of culturally-relevant modeling pedagogy.

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