MATH 111-01, Assignment 6

This assignment is to help you practice problems from chapters 8 and 9 for the final exam. You do not have to turn in any solutions.

1. You wish to test $H_0 : \mu = \mu_0$ for some value μ_0. In the following situations, decide whether you should use a z-test (based on the standard normal distribution) or a t-test (based on the t-distribution) or if neither applies.

(a) You have five measurements from a normal distribution with known variance $\sigma^2 = 25$.
(b) You have five measurements with known variance $\sigma^2 = 100$ and do not know anything about the distribution.
(c) You have one hundred measurements from a normal distribution with unknown variance.
(d) You have ten measurements from a normal distribution with unknown variance.
(e) You have two measurements from a normal distribution with known variance $\sigma^2 = 1,000$.
(f) You have fifty measurements from a uniform distribution with unknown variance.
(g) You have two measurements from an unknown distribution with known variance $\sigma^2 = 10$.
(h) You have fifteen measurements from a normal distribution and don't know anything about the variance.
(i) You have twelve measurements from a distribution that is not normal.

2. State H_0 and H_a in the following cases. It could be one-sample or two-sample problems. In the two-sample problems, state if you should compare independent populations or paired differences.

(a) You want to prove that a certain coin is unfair.
(b) You want to investigate if a certain die gives sixes less often than it should.
(c) You want to investigate if there is a difference in blood pressure levels between city people and country folks.
(d) You want to prove that a certain type of lightbulb has mean lifetime less than 1,500 hours.
(e) You want to prove that coffee increases IQ by at least 10 units.

(f) You want to investigate if your favorite politician has at least 50% support in the population.
(g) You want to prove that your favorite politician's support has increased since last year.

3. You do a hypothesis test of H_0 vs H_a on significance level 5%. Which of the following statements are true?

(a) The probability that H_0 is true is 5%
(b) The probability that you reject H_0 is 5%
(c) The probability that you incorrectly reject H_0 is 5%
(d) The probability that you reject H_0 when it is true is 5%
(e) The probability that you reject H_0 when it is false is 95%
(f) The probability that you fail to reject H_0 is 5%
(g) The probability that you fail to reject H_0 when it is false is 5%

4. You do a test of H_0 vs H_a and get p-value 0.03. Which of the following statements are true?

(a) You can reject H_0 on the 1% level
(b) You can reject H_0 on the 5% level
(c) You can not reject H_0
(d) There is no evidence of favor of H_a
(e) You can reject H_0 on any level smaller than 3%
(f) You can reject H_0 on any level larger than 3%

5. Find confidence intervals for the following data sets with the specified confidence levels. Use the calculator!

(a) 880 Swedes are asked if they like Princess Madeleine (www.princess-madeleine.com) and 619 answer yes. Find a 95% confidence interval for the proportion of the population that like her.
(b) 15 Norwegian salmon are weighed giving a sample mean of 40.5 pounds and a sample variance of 23.1. Find a 99% confidence interval for the mean weight μ. Weights follow a normal distribution.
(c) Diameters of 12 ball bearings are measured and give a sample mean of 12.1 grams. Find a 95% confidence interval for the mean diameter. Diameters follow a normal distribution with $\sigma = 0.05$.
(d) The difference in length between two types of snakes is investigated. A

sample of eight snakes of type A gave sample mean 35 and sample variance 9, a sample of seven snakes of type B gave sample mean 31 and sample variance 11. Find a 90% confidence interval for the difference between the two mean lengths. Lengths of both types follow normal distributions with the same variance σ^2.
(e) Blood pressure was measured on five patients before and after medication. The "before values" were: 134, 123, 144, 145, 171 and the "after values": 130, 111, 136, 130, 149 (same order). Find a 95% confidence interval for the difference between before and after medication. Data are normal.

6. In each of the following cases, state the null and alternative hypothesis, test if the null hypothesis can be rejected on the given significance level, and find the p-value. Use the calculator!

(a) A type of lightbulb is claimed to last for more than one thousand hours. Ten lifetimes are measured: 803, 884, 901, 1003, 1010, 1055, 1080, 1100, 1103, 1156. Data are normal with $\sigma = 100$. Test the claim on the 5% level.
(b) Striped bass in a certain river used to have a mean weight of 10 pounds. To test if this has changed, 12 bass are caught and weighed giving a sample mean of 13.1 and a sample standard deviation of 1.9. Weights follow a normal distribution. Test if there has been a change on level 5%.
(c) A politician claims to have support from more than half of the voters. He bases this on an opinion poll where 523 out of a thousand voters gave him support. Test his claim on the 1% level.
(d) The same politician claims that his support has increased between two polls taken a year apart. The first poll gave him support from 420 out of 900, the second poll 523 out of 1000. Test his claim on the 1% level.
(e) IQs in two populations are compared. The two sample means are 123 and 109 and both samples have size 4. Data are normal with $\sigma_1 = \sigma_2 = 15$. Test if there is a difference on level 5%.
(f) For the blood pressure data in 5(e) above, test on the 1% level if the medicine lowers blood pressure. Also test on the 1% level if the difference is at least five units.

ANSWERS.

1(a) z-test **(b)** neither **(c)** z-test **(d)** t-test **(e)** z-test **(f)** z-test **(g)** neither **(h)** t-test **(i)** neither

2(a) $H_0 : p = 0.5$ vs $H_a : p \neq 0.5$
(b) $H_0 : p = 1/6$ vs $H_a : p < 1/6$
(c) $H_0 : \mu_1 - \mu_2 = 0$ vs $H_a : \mu_1 - \mu_2 \neq 0$, independent populations
(d) $H_0 : \mu = 1500$ vs $H_a : \mu < 1500$
(e) $H_0 : \mu_D = \mu_1 - \mu_2 = 10$ vs $H_a : \mu_D = \mu_1 - \mu_2 > 10$, paired differences
(f) $H_0 : p = 0.5$ vs $H_a : p > 0.5$
(g) $H_0 : p_1 = p_2$ vs $H_a : p_1 > p_2$, independent populations

3. (c) and (d) are true. The significance level is the probability that you reject H_0 when it is in fact true. You can *never* compute the probability that H_0 is true or false; this we simply do not know.

4. (b) and (f) are true. The p-value is the *smallest* significance level for which you can *reject* H_0. The smaller the p-value the stronger the evidence in favor of H_a against H_0.

5(a) $p = 0.70 \pm 0.03(95\%)$
(b) $\mu = 40.5 \pm 3.7$ (99%)
(c) $\mu = 12.1 \pm 0.03$ (95%)
(d) $\mu_1 - \mu_2 = 4 \pm 2.9$ (90%)
(e) $\mu_D = \mu_1 - \mu_2 = 12.2 \pm 8.5$ (95%)

6(a) $H_0 : \mu = 1000$ vs $H_a : \mu > 1000$. Can not reject H_0, p-value = 0.38.
(b) $H_0 : \mu = 10$ vs $H_a : \mu \neq 10$. Reject H_0, p-value 0.00015.
(c) $H_0 : p = 0.5$ vs $H_a : p > 0.5$. Can not reject H_0 (no evidence for his claim), p-value = 0.07.
(d) $H_0 : p_2 = p_1$ vs $H_a : p_2 > p_1$. Reject H_0, p-value = 0.007.
(e) $H_0 : \mu_1 - \mu_2 = 0$ vs $H_a : \mu_1 - \mu_2 \neq 0$. Can not reject H_0, p-value 0.19.
(f) $H_0 : \mu_D = \mu_1 - \mu_2 = 0$ vs $H_a : \mu_D > 0$. Reject H_0, p-value = 0.008.
$H_0 : \mu_D = 5$ vs $H_a : \mu_D > 5$. Can not reject H_0, p-value = 0.04.