MATH 111-01, Assignment 5, complete before April 7

This assignment is to help you prepare for the test April 7. You do not have to turn in any solutions.

Continuous Random Variables (Chapter 5)

Note: For *continuous* random variables, it does not matter if we use strict or nonstrict inequalities. Thus, $P(x \leq t)$ is always the same as $P(x < t)$, $P(x \geq t)$ always the same as $P(x > t)$ and so on. Remember that the probabilities are areas under the graph of the pdf so a single point will not make a difference. **Always keep this in mind!** (Why is the same not true for discrete random variables?)

1. Let $x \sim$ unif[0,1] and find the distribution of **(a)** $-x$ **(b)** $1-x$ **(c)** $x-1$ **(d)** $2x+5$ **(e)** $2x-5$ **(f)** $5x-2$ **(g)** $(5x+1)/3$

2. Let $x \sim$ unif[10,20] and find **(a)** $P(x \leq 15)$ **(b)** $P(x < 18)$ **(c)** $P(x > 18)$ **(d)** $P(12 < x < 15)$ **(e)** $P(x \leq 9)$ **(f)** $P(x < 11$ or $x > 18)$ **(g)** $P(|x-15| > 4)$

3. Let $x \sim$ unif$[a,b]$ where x has mean μ and variance σ^2. Find a and b if **(a)** $\mu = 1/2, \sigma^2 = 1/12$ **(b)** $\mu = 1, \sigma^2 = 1/12$ **(c)** $\mu = 1, \sigma^2 = 1/3$ **(d)** $\mu = 2, \sigma^2 = 4/3$ **(e)** $\mu = 0, \sigma^2 = 3$

4. Let x have pdf $f(x) = ax, 0 \leq x \leq 2$. Find **(a)** the value of a **(b)** $P(x \leq 1)$ **(c)** $P(x > 1.5)$

5. Let x have pdf $f(x) = 1 - x/2, 0 \leq x \leq 2$. Sketch the graph of $f(x)$ and find **(a)** $P(x \geq 1)$ **(b)** $P(0.5 \leq x \leq 1)$

6. Let $x \sim N(0,1)$. What is the distribution of $-x$?

7. Let $x \sim N(4,4)$ $(\sigma^2 = 4)$ and find **(a)** $P(x \leq 6)$ **(b)** $P(x \leq 4)$ **(c)** $P(x > 4)$ **(d)** $P(x < 5.5)$ **(e)**$P(4.5 \leq x \leq 6)$ **(f)** $P(|x-5| \leq 1)$

8. In problem 7, find the value t which is such that **(a)** $P(x \leq t) = 0.95$

(b) $P(x \geq t) = 0.01$ **(c)** $P(x \leq t) = 0.8)$ **(d)** $P(x \leq t) = 0.05)$ **(e)** $P(4 - t \leq x \leq 4 + t) = 0.95$

Point Estimators, Sampling Distributions, and the Central Limit Theorem (Chapter 6)

From the book, Chapter 6: 10, 11, 27, 48

9. Let $x_1, ..., x_n$ be observations from a uniform distribution on $[0, 1]$. What is the approximate distribution of $\bar{x}$ if **(a)** $n = 20$ **(b)** $n = 100$?

10. Let $x_1, ..., x_{20}$ be twenty observations from a distribution that has $\mu = 5$ and $\sigma^2 = 5$. Find the approximate probabilities **(a)** $P(\bar{x} \leq 5.5)$ **(b)** $P(\bar{x} > 6)$.

Confidence Intervals (Chapter 7)

From the book, Chapter 7: 7–12, 28, 31, 32, 35, 42, 43, 45, 46, 57, 63, 64, 75

11. Twenty sea bass are caught and weighed, giving a sample mean of 8. The variance is known to be 4. Give a confidence interval for the mean weight μ with confidence level **(a)** 99% **(b)** 95% **(c)** 90%.

12. Metal rods are being manufactured and are supposed to have a length of 10 inches. The following is a sample of eight measured lengths: 11.1, 8.7 12.3 12.0 9.7 13.8 14.1 11.9. Find a 95% confidence interval for the mean length μ. Do you think that the manufacturing process is working as it should?