

MATH 301/601, Review problems, 2007

1. You are dealt a bridge hand (13 cards from a regular deck of 52). Find a combinatorial expression for the probability that you get 6 hearts, 3 diamonds, and the remaining 4 cards all in the same black suit.
2. You are dealt a poker hand. Below is an expression for the probability to get one pair but one factor X is missing. What is X ?

$$\frac{13 \cdot \binom{4}{2} \cdot X \cdot 4 \cdot 4 \cdot 4}{\binom{52}{5}}$$

3. You test for a disease that about one in 500 people have. If you have the disease the test is always positive. If you do not have the disease the test is 95% accurate.

- (a) What is the probability that you test positive?
- (b) If you test positive, what is the probability that you have the disease?

4. True or false?

- (a) If $X \sim \text{bin}(n, p)$, then $2X \sim \text{bin}(2n, p)$
- (b) If $X \sim \text{Poi}(\mu)$, then $2X \sim \text{Poi}(2\mu)$
- (c) If $X \sim \text{unif}[0, 1]$, then $2X \sim \text{unif}[0, 2]$
- (d) If $X \sim \text{exp}(\lambda)$, then $2X \sim \text{exp}(\lambda/2)$

5. Suppose that you guess at random at (a)-(d) in the Problem 4 and let X be the number of correct answers.

- (a) What is the distribution of X ?
- (b) What is the expected number of correct answers?
- (c) What is the probability that you get all 4 correct answers?
- (d) Now suppose that 100 students do Problem 4 by guessing at random. Let Y be the number of students who get all 4 correct answers. What is the distribution and expected value of Y ?

6. The continuous random variable X has range $[0, 2]$ and pdf

$$f(x) = ax^2, \quad 0 \leq x \leq 2$$

- (a) Find the constant a .
- (b) Find the median m of X (the point m that has $P(X \leq m) = 1/2$).

7. Let $X \sim \text{unif}[0,2]$ and let V be the volume of a cube with side X . Find

- (a) the pdf of V (including the range)
- (b) $E[V]$ and $\text{Var}[V]$.

8. In the month of March, on average 4 tornadoes are registered in the state of Oklahoma. Compute the probability of the following events. State what assumption you are making about the occurrence of tornadoes.

- (a) Exactly 3 tornadoes are registered next March.
- (b) At least one tornado is registered next March.
- (c) There are no tornadoes registered during the last week (7 days) of next March.

9. Let X and Y be independent continuous random variables. Which of the following statements are always true?

- (a) $f(x, y) = f_X(x)f_Y(y)$
- (b) $E[XY] = E[X]E[Y]$
- (c) $E[X^2Y^2] = E[X^2]E[Y^2]$
- (d) $\text{Var}[XY] = \text{Var}[X]\text{Var}[Y]$

10. Let $X \sim N(0, 1)$ and let $Y = X^2$. Find

- (a) $P(X > 1)$
- (b) $P(X = 1)$
- (c) $P(-2 \leq X \leq 1)$
- (d) $P(Y > 1)$.
- (e) the pdf of Y expressed in terms of $\varphi(x)$
- (f) $P(X < 1, Y < 4)$

11. Let $X \sim \text{unif}[0, 1]$ and let the conditional pdf of Y given $X = x$ be uniform on $[0, 2x]$.

- (a) Find the joint pdf of (X, Y) , $f(x, y)$.
- (b) Find $E[Y|X = x]$ and use it to compute $E[Y]$.

12. Let (X, Y) be uniform on the triangle with corners in $(0, 0)$, $(2, 0)$, and $(2, 1)$.

- (a) Find the joint pdf of (X, Y) .
- (b) Find the marginal pdf of X .
- (c) Find the conditional pdf of Y given $X = x$. Is this a uniform distribution?

13. Let X and Y be independent and $\exp(1)$. Find

- (a) the joint pdf of (X, Y) including range
- (b) $P(Y \leq 2X)$
- (c) the pdf of $\min(X, Y)$. What is this distribution called?

14. Let the random variable X have pdf $f(x) = xe^{-x^2/2}$, $x \geq 0$. Show how you can simulate an observation on X based on $U \sim \text{unif}[0, 1]$.

15. Let X_1, \dots, X_n be a sample from a distribution with unknown variance σ^2 . Find the method of moments estimator of σ^2 .

16. Let X_1, \dots, X_n be a sample from a distribution with pdf $f(x) = a^2xe^{-ax}$, $x \geq 0$. Find the maximum likelihood estimator of a .

17. Let X_1, \dots, X_n be a sample from the pdf $f(x) = (a + 1)x^a$, $0 \leq x \leq 1$ where a is an unknown parameter. Find the method of moments and maximum likelihood estimators of a .