MATH 301, simulation exercises

The following is a brief description of how to use Matlab for simulations. If you prefer, you may use other software that can do the same. Matlab is available on the server treat.tcs.tulane.edu, started with the command 'matlab'.

Most mathematical software is able to simulate observations from many distributions. In Matlab this is done with the 'random' command. For example, the command 'x=random('unif',0,1,1,1000)' generates 1000 observations (i.e. observations of $X_1, X_2, ..., X_{1000}$ where the X_k are i.i.d.) from a uniform distribution on (0,1) and saves them in the vector x. Do 'help random' to see how it works and which distributions that are supported.

By using the 'hist' command, you can plot so called histograms for your observations. Histograms are obtained by dividing the range into suitably sized intervals and then plotting the number of observations for each such interval. The histogram gives an idea of the shape of the pdf or pmf the observations come from. Note that Matlab allows you to choose the number of such intervals (called 'bins'). The default is 10 but larger values may sometimes be more instructive.

1. For each of the following distributions, simulate a large number of observations and compare the shape of the pdf and the shape of the histogram.

a. Uniform distribution on (a, b) for some different choices of a and b.

b. Exponential distribution for some different choices of the parameter a.

c. Normal distribution for some different choices of μ and σ^2.

d. Binomial distribution for some different choices of n and p.

2. Let $X \sim$ unif $[0, 1]$ and let $Y|X = x \sim$ unif$[0, x]$ (example done in class). You can simulate n observations on (X, Y) by the command 'x=unif(0,1,1,n)' followed by 'y=unif(0,x,1,n)' (note that x is a vector). They can then be plot-

ted by the command 'plot(x,y,'.')'. Do this and note how the points are not uniformly spread over the triangle.

3. Do the previous problem again with the only difference that X is chosen according to the pdf $f_X(x) = 2x, 0 \leq x \leq 1$. Use the inverse transformation method to figure out how to simulate X from $U \sim \text{unif}[0, 1]$. Note how the points now are uniform on the triangle. (The Matlab command for $\sqrt{x}$ is 'sqrt(x)' which also works if x is a vector.)

4. Simulation can be a very useful method in finding means and variances that are impossible or hard to compute explicitly. As an example, consider the following problem, called the "gambler's ruin". Two players, A and B play a game where a fair coin is flipped. If it shows heads, A pays B one dollar and if it shows tails, B pays A one dollar. Suppose that A starts with a dollars and B with b dollars (both a and b are integers) and let T be the time when one of them is ruined, that is, T is the gambling round after which either A or B has no money left. There is a simple explicit formula for the mean $E[T]$, expressed in terms of a and b, but it is difficult to find it without invoking more advanced theory than we have done in this course (just try and you will see!). Instead, run simulations of the game and try to guess the expression for $E[T]$ based on your simulations.