1 Plug in the values $x = 0$ and $x = 1$ to get the range of the new random variable and watch out for minus signs. **(a)** Here $x = 0$ gives $-x = 0$ and $x = 1$ gives $-x = -1$. The range of $-x$ is -1 to 0 so it is unif[-1,0] **(b)** Here $x = 0$ gives $1 - x = 1$ and $x = 1$ gives $1 - x = 0$. Hence $1 - x$ is unif[0,1] **(b)** unif[0,1] **(c)** unif[-1,0] **(d)** unif[5,7] **(e)** unif[-5,-3] **(f)** unif[-2,3] **(g)** unif[1/3,2]

2. In general if $x \sim$ unif$[c, d]$, then $P(x \leq t) = (t - c)/(d - c)$, $P(x > t) = (d - t)/(d - c)$, and $P(a \leq x \leq b) = (b - a)/(d - c)$ for t, a, and b inside the range $[c, d]$. We get **(a)** 0.5 **(b)** 0.8 **(c)** 0.2 **(d)** 0.3 **(e)** 0 **(f)** 0.3 **(g)** 0.2

3. In general, $\mu = (a + b)/2$ and $\sigma^2 = (b - a)^2/12$. In each case, solve for a and b to get **(a)** $a = 0, b = 1$ **(b)** $a = 1/2, b = 3/2$ **(c)** $a = 0, b = 2$ **(d)** $a = 0, b = 4$ **(e)** $a = -3, b = 3$

4. **(a)** The region under the graph of the pdf is a triangle with area $2 \cdot 2a/2 = 2a$ and since this must equal one, we get $a = 1/2$. **(b)** $P(x \leq 1) = 1/4$ (area of triangle under the graph to the left of 1) **(c)** $P(x > 1.5) = 1 - (x \leq 1.5) = 1 - (1.5 \cdot (1.5/2)/2 = 0.4375$

5. **(a)** $P(x \leq 1) = 3/4$ (one minus the area of the triangle to the right of 1) **(b)** $P(0.5 \leq x \leq 1) = 0.3125$ (the difference between the areas of two triangles)

6. Also $N(0, 1)$ since the distribution is symmetric around 0. Thus, x and $-x$ have the same *distribution* but they will always get different *values*. Think about it (and compare with problem 1b above).

7. **(a)** $P(x \leq 6) = \Phi((6 - 4)/2) = \Phi(1) = 0.84$ **(b)** $\Phi(-1) = 1 - \Phi(1) = 0.16$ **(c)** $1 - \Phi(0) = 0.5$ **(d)** $\Phi(0.75) = 0.77$ **(e)** $\Phi(1) - \Phi(0.25) = 0.24$ **(f)** $\Phi(1) - \Phi(-1) = 0.68$

8. **(a)** $P(x \leq t) = \Phi((t - 4)/2) = 0.95$ gives $(t - 4)/2 = 1.64$ which gives $t = 5.28$ **(b)** $P(x \geq t) = 1 - \Phi((t - 4)/2) = 0.01$ gives $(t - 4)/2 = 1.96$ which gives $t = 7.92$ **(c)** $\Phi((t - 4)/2) = 0.8$ gives $(t - 4)/2 = 0.84$ which gives $t = 5.68$ **(d)** $\Phi((t - 4)/2) = 0.05$ gives $\Phi((4 - t)/2) = 0.95$ which gives $(4 - t)/2 = 1.64$ which gives $t = 0.72$ **(e)** $\Phi(t/2) - \Phi(-t/2) = 2\Phi(t/2) - 1 = 0.95$ gives $t/2 = 1.96$ which gives $t = 3.92$

9. In general, $\bar{x}$ is approximately $N(\mu, \sigma^2/n)$. Here, $\mu = 1/2$ and $\sigma^2 = 1/12$ which gives **(a)** $N(1/2, 1/240)$ **(b)** $N(1/2, 1/1200)$

10. Here $\sigma^2/n = 5/20 = 1/4$ so $\bar{x}$ is approximately $N(5, 1/4)$ iand since $\sqrt{1/4} = 1/2$ we get **(a)** $P(\bar{x} \leq 5.5) \approx \Phi((5.5-5)/(1/2)) = \Phi(1) = 0.84$ **(b)** $P(\bar{x} > 6) = 1 - P(\bar{x} \leq 6) \approx 1 - \Phi(2) = 0.03$

11. The general form is $\mu = \bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n}\ (1-\alpha)$. We have $\bar{x} = 8, n = 20$, and $\sigma = 2$. **(a)** $1 - \alpha = 0.99$ gives $\alpha/2 = 0.005$ and $z_{0.005} = 2.58(\Phi(2.58) = 0.995)$. We get $\mu = 8 \pm 1.15$ (99%). **(b)** $z_{0.025} = 1.96$, $\mu = 8 \pm 0.88$ (95%) **(c)** $z_{0.05} = 1.64$, $\mu = 8 \pm 0.73(90\%)$

12. The general interval is $\mu = \bar{x} \pm t_{\alpha/2}s/\sqrt{n}\ (1-\alpha)$. We have $n = 8$, $\bar{x} = 11.7$, $s = 1.85$, and with confidence level $1 - \alpha = 0.95$ we get $t_{0.025} = 2.365(df = n - 1 = 7, \alpha/2 = 0.025)$ which gives $\nu = 11.7 \pm 2.365 \cdot 1.85/\sqrt{8} = 11.7 \pm 1.5(95\%)$. Since this interval does not contain the target value of 10, the process probably needs to be adjusted.