INDEED SHALOSHABLE!

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In an article posted at the `sci.math.research` newsgroup and found at `gopher://davinci.lfc.edu:70/0R373926-376715/MathRelItems/scimathArchive/scimathres.archive`, it was mentioned that a certain identity was seemingly not provable by Ekhad. Here, we shall refute this and actually demonstrate how much *shaloshable* it is!

CLAIM: The alleged identity

$$\frac{1}{4}+\sum_{n=0}^{\infty}\binom{2n-1}{n}^2\frac{1}{2^{4n}(n+1)}=\frac{1}{\pi} \tag{1}$$

is indeed shaloshable!!

Proof: Denote $(a)_k := a(a+1)\cdots(a+k-1)$, then we have

$$s(n):=\sum_k\frac{(1/2)_k(-n)_k(n+1)(n+3/4)!(n+1/4)!}{k!(3/2+n)_k(2n+2k+3)(n+1/2)!^2}\equiv \text{CONSTANT}. \tag{2}$$

The Maple package *EKHAD* supplies the recurrence $s(n+1)-s(n)=0$ and a WZ "certificate",

$$\frac{-k(3n+4+4nk+6k)}{4(n-k+1)(2n+3)(n+1)}.$$

Check at, say $n=0$ and determine the constant, which is $\sqrt{2}/4$. To prove the claim, first rewrite equation (2) as

$$\sum_k\frac{(1/2)_k(-n)_k(n+1)}{k!(3/2+n)_k(2n+2k+3)}=\frac{\sqrt{2}}{4}\frac{(n+1/2)!^2}{(n+3/4)!(n+1/4)!}$$

and then "plug-in" $n=-1/2$. The rest is trivial. □

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