PROOF OF FORMULA 3.194.1

$$\int_0^a \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = \frac{a^\mu}{\mu}\,{}_2F_1\left(\begin{matrix}\nu & \mu \\ \mu+1 & \end{matrix}\middle| -ab\right)$$

The proof employs the basic integral representation of the hypergeometric function

$${}_2F_1\left(\begin{matrix}a & b \\ & c\end{matrix}\middle| z\right) = \frac{1}{B(b,c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}\,dt \qquad \text{for } \operatorname{Re} c > \operatorname{Re} b > 0,$$

which appears as entry **9.111**.

Let $x = at$ to obtain

$$\int_0^a \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = a^\mu \int_0^1 t^{\mu-1}(1+abz)^{-\nu}\,dt.$$

Then choose $a \mapsto \nu$, $b \mapsto \mu, c \mapsto 1+\mu$ and $z \mapsto -ab$ to obtain

$$\int_0^a \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = a^\mu B(\mu,1)\ {}_2F_1\left(\begin{matrix}\nu & \mu \\ 1+\mu & \end{matrix}\middle| -ab\right).$$

The result is simplified by using $B(\mu,1) = 1/\mu$.