PROOF OF FORMULA 3.272.1

$$\int_0^1 \frac{x^{n-1} + x^{n-1/2} - 2x^{2n-1}}{1-x}\, dx = 2\ln 2$$

Write the integral as

$$\int_0^1 \frac{x^{n-1} + x^{n-1/2} - 2x^{2n-1}}{1-x}\, dx = \int_0^1 \frac{x^{n-1} - x^{2n-1}}{1-x}\, dx + \int_0^1 \frac{x^{n-1/2} - x^{2n-1}}{1-x}\, dx.$$

Using entry **3.231.5**

$$\int_0^1 \frac{x^{\mu-1} - x^{\nu-1}}{1-x}\, dx = \psi(\nu) - \psi(\mu)$$

it follows that

$$\int_0^1 \frac{x^{n-1} + x^{n-1/2} - 2x^{2n-1}}{1-x}\, dx = 2\psi(2n) - \psi(n) - \psi(n+1/2).$$

The identity

$$\psi(2t) = \frac{1}{2}\left(\psi(t) + \psi(t+1/2)\right) + \ln 2$$

gives the result.