

PROOF OF FORMULA 3.311.4

$$\int_0^{\infty} \frac{e^{-qx} dx}{1 - ae^{-px}} = \sum_{k=0}^{\infty} \frac{a^k}{q + kp} = \frac{1}{q} {}_2F_1 \left[\frac{q}{p}, 1; 1 + \frac{q}{p}; a \right]$$

The expansion

$$\frac{1}{1 - ae^{-px}} = \sum_{k=0}^{\infty} a^k e^{-pkx}$$

is integrated term by term to produce the result.