PROOF OF FORMULA 3.317.2

$$\int_{-\infty}^{\infty}\left[\frac{1}{(1+e^{-x})^{\nu}}-\frac{1}{(1+e^{-x})^{\mu}}\right]dx=\psi(\mu)-\psi(\nu)$$

The change of variables $t=e^{-x}$ gives

$$\int_{-\infty}^{\infty}\left[\frac{1}{(1+e^{-x})^{\nu}}-\frac{1}{(1+e^{-x})^{\mu}}\right]dx=\int_{0}^{\infty}\left[\frac{1}{(1+t)^{\nu}}-\frac{1}{(1+t)^{\mu}}\right]\frac{dt}{t}.$$

The result now follows from the integral representation **8.361.2**:

$$\psi(a)=\int_{0}^{\infty}\left[e^{-t}-\frac{1}{(1+t)^{a}}\right]\frac{dt}{t}$$

for the digamma function ψ.