PROOF OF FORMULA 3.323.2

$$\int_{-\infty}^{\infty} e^{-p^2x^2 \pm qx} \, dx = \frac{\sqrt{\pi}}{p} e^{q^2/4p^2}$$

The case of the plus sign is discussed here, the minus sign is identical.

The change of variables $t = px$ gives

$$\int_{-\infty}^{\infty} e^{-p^2x^2+qx} \, dx = \frac{1}{p} \int_{-\infty}^{\infty} e^{-t^2+qt/p} \, dt.$$

Completing squares in the exponent yields

$$\frac{1}{p} \int_{-\infty}^{\infty} e^{-t^2+qt/p} \, dt = \frac{1}{p} e^{q^2/4p^2} \int_{-\infty}^{\infty} e^{-(t+q/2p)^2} \, dt.$$

The result follows from the value of the normal integral

$$\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}.$$