PROOF OF FORMULA 3.333.1

$$\int_{-\infty}^{\infty} \frac{e^{-sx}\,dx}{\exp(e^{-x})-1} = \Gamma(s)\zeta(s)$$

Let $t = e^{-x}$ to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-sx}\,dx}{\exp(e^{-x})-1} = \int_{0}^{\infty} \frac{t^{s-1}\,dt}{e^{t}-1}.$$

This integral is evaluated in entry **3.411.1** and it has value $\Gamma(s)\zeta(s)$.