PROOF OF FORMULA 3.333.2

$$\int_{-\infty}^{\infty} \frac{e^{-sx}\,dx}{\exp(e^{-x})+1} = \begin{cases} (1-2^{1-s})\Gamma(s)\zeta(s) & \text{if } s \neq 1 \\ \\ \ln 2 & \text{if } s = 1 \end{cases}$$

Let $t = e^{-x}$. Then

$$\int_{-\infty}^{\infty} \frac{e^{-sx}\,dx}{\exp(e^{-x})+1} = \int_0^{\infty} \frac{t^{s-1}\,dt}{e^t+1}.$$

For $s \neq 1$ the value is computed in formula **3.411.3**. For $s = 1$, the change of variables $y = e^t + 1$ gives

$$\int_0^{\infty} \frac{dt}{e^t+1} = \int_2^{\infty} \frac{dy}{y(y-1)}.$$

The partial fraction decomposition

$$\frac{1}{y(y-1)} = \frac{1}{y-1} - \frac{1}{y}$$

gives the result.