PROOF OF FORMULA 3.351.2

$$\int_a^\infty x^n e^{-\mu x}\,dx = \frac{\Gamma(n+1, a\mu)}{\mu^{n+1}} = e^{-a\mu}\sum_{k=0}^{\infty}\frac{n!}{k!}\frac{a^k}{\mu^{n-k+1}}$$

Formula **2.321.2** states that

$$\int x^n e^{-\mu x}\,dx = -e^{-\mu x}\left(\sum_{k=0}^{n}\frac{k!\binom{n}{k}}{\mu^{k+1}}x^{n-k}\right).$$

Therefore

$$\int_a^\infty x^n e^{-\mu x}\,dx = e^{-\mu a}\sum_{k=0}^{n}\frac{n!}{k!}\frac{a^k}{\mu^{n-k+1}}.$$

The incomplete gama function is defined in **8.350.2** by

$$\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1}e^{-t}\,dt,$$

and the change of variables $t = \mu x$ provides the evaluation

$$\int_a^\infty x^n e^{-\mu x}\,dx = \frac{\Gamma(n+1, a\mu)}{\mu^{n+1}}.$$

This gives the result.