PROOF OF FORMULA 3.366.2

$$\int_0^\infty \frac{(x+b)e^{-\mu x}\,dx}{\sqrt{x^2+2bx}} = be^{b\mu}K_1(b\mu)$$

The integral representation for the Bessel function K_ν appears in **8.432.3**:

$$K_\nu(z) = \frac{z^\nu\sqrt{\pi}}{2^\nu\Gamma(\nu+1/2)}\int_1^\infty e^{-tz}(t^2-1)^{\nu-1/2}.$$

In particular

$$K_1(z) = z\int_1^\infty e^{-zt}\sqrt{t^2-1}\,dt.$$

Complete the square to get

$$\int_0^\infty \frac{(x+b)e^{-\mu x}\,dx}{\sqrt{x^2+2bx}} = \int_0^\infty \frac{(x+b)e^{-\mu x}\,dx}{\sqrt{(x+b)^2-b^2}}.$$

The change of variables $s = x+b$ gives

$$\int_0^\infty \frac{(x+b)e^{-\mu x}\,dx}{\sqrt{(x+b)^2-b^2}} = e^{b\mu}\int_b^\infty \frac{se^{-\mu s}\,ds}{\sqrt{s^2-b^2}}.$$

Now let $r = bs$ to obtain the result.