PROOF OF FORMULA 3.372

$$\int_0^\infty x^{n-1/2}(x+2)^{n-1/2}e^{-px}dx = \frac{(2n-1)!!}{p^n}e^p K_n(p)$$

Start with

$$\int_0^\infty x^{n-1/2}(x+2)^{n-1/2}e^{-px}dx = \int_0^\infty \left[(x+1)^2-1\right]^{n-1/2} e^{-px}dx,$$

and make the change of variables $t = x+1$ to obtain

$$\int_0^\infty \left[(x+1)^2-1\right]^{n-1/2} e^{-px}dx = e^p \int_1^\infty (t^2-1)^{n-1/2}e^{-pt}\, dt.$$

The integral representation

$$K_\nu(z) = \frac{\Gamma(\frac{1}{2})}{\Gamma(\nu+\frac{1}{2})}\left(\frac{z}{2}\right)^\nu \int_1^\infty e^{-zt}(t^2-1)^{\nu-\frac{1}{2}}\, dt,$$

gives the result.