PROOF OF FORMULA 3.411.16

$$\int_{-\infty}^{\infty} \frac{x^2 e^{-\mu x}}{1+e^{-x}}\, dx = \frac{\pi^3[2-\sin^2(\pi\mu)]}{\sin^3(\pi\mu)}$$

The change of variables $t = e^{-x}$ gives

$$\int_{-\infty}^{\infty} \frac{x^2 e^{-\mu x}}{1+e^{-x}}\, dx = \int_0^{\infty} \frac{t^{\mu-1}\ln^2 t\, dt}{1+t}.$$

The formula

$$\int_0^{\infty} \frac{t^{\mu-1}\, dt}{1+t} = \frac{\pi}{\sin \pi\mu}$$

follows from the integral representation for the beta function

$$B(a,b) = \int_0^{\infty} \frac{t^{a-1}\, dt}{(1+t)^{a+b}}$$

and the identity $B(a, 1-a) = \pi/\sin \pi a$.

The original integral is obtained by differentiating twice with respect to the parameter μ.