# PROOF OF FORMULA 3.411.22

$$\int_0^\infty \frac{x^{p-1}\,dx}{e^{rx}-q} = \frac{\Gamma(p)}{qr^p}\sum_{k=1}^\infty \frac{q^k}{k^p}$$

Expand the integrand as

$$\frac{1}{e^{rx}-q} = e^{-rx}\sum_{j=0}^\infty q^j e^{-rxj},$$

and integrate to produce

$$\int_0^\infty \frac{x^{p-1}\,dx}{e^{rx}-q} = \sum_{j=0}^\infty q^j \int_0^\infty x^{p-1}e^{-(j+1)rx}\,dx.$$

The change of variables $t=(j+1)rx$ gives

$$\int_0^\infty \frac{x^{p-1}\,dx}{e^{rx}-q} = \sum_{j=0}^\infty \frac{q^j}{(j+1)^p r^p}\int_0^\infty t^{p-1}e^{-t}\,dt.$$

The last integral is recognized as $\Gamma(p)$.