PROOF OF FORMULA 3.411.23

$$\int_{-\infty}^{\infty} \frac{xe^{\mu x}\,dx}{b+e^x} = \frac{\pi\, b^{\mu-1}}{\sin \pi\mu} \left[\ln b - \pi \cot \pi\mu\right]$$

The change of variables $x = t + \ln b$ yields $e^x = be^t$ and

$$\int_{-\infty}^{\infty} \frac{xe^{\mu x}\,dx}{b+e^x} = \ln b \int_{-\infty}^{\infty} \frac{e^{\mu t}\,dt}{1+e^t} + \int_{-\infty}^{\infty} \frac{te^{\mu t}\,dt}{1+e^t}.$$

The change of variables $s = e^t$ shows that the first integral is

$$\int_{-\infty}^{\infty} \frac{e^{\mu t}\,dt}{1+e^t} = \int_0^{\infty} \frac{s^{\mu-1}\,ds}{1+s} = B(\mu, 1-\mu) = \frac{\pi}{\sin \pi\mu}.$$

The value of the second integral comes from differetiating the first one with respect to μ:

$$\int_{-\infty}^{\infty} \frac{te^{\mu t}\,dt}{1+e^{-t}} = -\frac{\pi^2 \cot \pi\mu}{\sin \pi\mu}.$$

Replacing, produces the result.