PROOF OF FORMULA 3.431.1

$$\int_0^\infty \left(e^{-\mu x} - 1 + \mu x - \tfrac{1}{2}\mu^2 x^2\right) x^{\nu-1}\, dx = -\frac{\Gamma(\nu+3)}{\nu(\nu+1)(\nu+2)\mu^\nu}$$

Integrate by parts to produce

$$
\begin{aligned}
\int_0^\infty \left(e^{-\mu x} - 1 + \mu x - \tfrac{1}{2}\mu^2 x^2\right) x^{\nu-1}\, dx &= -\frac{\mu}{\nu}\int_0^\infty \left(e^{-\mu x} - 1 + \mu x\right) x^\nu\, dx \\
&= -\frac{\mu^2}{\nu(\nu+1)}\int_0^\infty \left(e^{-\mu x} - 1\right) x^{\nu+1}\, dx \\
&= -\frac{\mu^3}{\nu(\nu+1)(\nu+2)}\int_0^\infty e^{-\mu x} x^{\nu+2}\, dx
\end{aligned}
$$

and then let $t = \mu x$ to obtain the result.