PROOF OF FORMULA 3.433

$$\int_0^\infty x^{p-1}\left[e^{-x}+\sum_{k=1}^{n}(-1)^k\frac{x^{k-1}}{(k-1)!}\right]dx=\Gamma(p)$$

Define

$$I_n(p):=\int_0^\infty x^{p-1}\left[e^{-x}+\sum_{k=1}^{n}(-1)^k\frac{x^{k-1}}{(k-1)!}\right]dx.$$

Integrate by parts to obtain

$$\begin{aligned}
I_n(p) &= -\int_0^\infty \frac{x^p}{p}\left[-e^{-x}+\sum_{k=2}^{n}(-1)^k\frac{x^{k-2}\,(k-1)}{(k-1)!}\right]dx\\
&= \frac{1}{p}\int_0^\infty x^p\left[e^{-x}-\sum_{k=1}^{n-1}(-1)^k\frac{x^{k-1}}{(k-1)!}\right]dx\\
&= \frac{1}{p}I_{n-1}(p+1).
\end{aligned}$$

The result now follows by induction of n.