FORMULA 3.462.5

$$\int_0^\infty xe^{-\mu x^2-2\nu x}\,dx = \frac{1}{2\mu} - \frac{\nu}{2\mu}\sqrt{\frac{\pi}{\mu}}e^{\nu^2/\mu}\left[1-\operatorname{erf}\left(\frac{\nu}{\sqrt{\mu}}\right)\right]$$

Let $t=\sqrt{\mu}x$ to obtain

$$\int_0^\infty xe^{-\mu x^2-2\nu x}\,dx = \frac{1}{\mu}\int_0^\infty te^{-t^2-2\alpha t}\,dt$$

with $\alpha=\nu/\sqrt{\mu}$. The change of variable $s=t+\alpha$ gives

$$\int_0^\infty xe^{-\mu x^2-2\nu x}\,dx = \frac{1}{\mu}e^{\alpha^2}\int_\alpha^\infty (s-\alpha)e^{-s^2}ds.$$

Then

$$\int_\alpha^\infty (s-\alpha)e^{-s^2}ds = \int_\alpha^\infty se^{-s^2}ds - \alpha\int_\alpha^\infty e^{-s^2}ds.$$

The first integral is $\frac{1}{2}e^{-\alpha^2}$ and the second one is written as

$$\int_\alpha^\infty e^{-s^2}ds = \frac{\sqrt{\pi}}{2} - \int_0^\alpha e^{-s^2}ds = \frac{\sqrt{\pi}}{2}\left(1-\operatorname{erf}(\alpha)\right).$$