PROOF OF FORMULA 3.466.1

$$\int_0^\infty \frac{e^{-\mu^2 x^2}\, dx}{x^2+b^2} = (1-\text{erf}(b\mu))\,\frac{\pi}{2b} e^{b^2\mu^2}$$

Let $x = bt$ and replace $b\mu$ by a to convert the formula into

$$\int_0^\infty \frac{e^{-a^2 t^2}\, dx}{1+t^2} = (1-\text{erf}(a))\,\frac{\pi}{2} e^{a^2}.$$

Define

$$f(a) := \int_0^\infty \frac{e^{-a^2 t^2}\, dx}{1+t^2},$$

and observe that

$$f'(a) = -2a \int_0^\infty e^{-a^2(t^2+1)}\, dt = -\sqrt{\pi} e^{-a^2}.$$

The initial value $f(0) = \pi/2$ and the definition

$$\text{erf}(s) = \frac{2}{\sqrt{\pi}} \int_0^s e^{-t^2}\, dt,$$

give the result.