PROOF OF FORMULA 3.466.2

$$\int_0^\infty \frac{x^2 e^{-\mu^2 x^2}\,dx}{x^2+b^2} = \frac{\sqrt{\pi}}{2\mu} - \frac{\pi b}{2} e^{b^2\mu^2}\left[1-\text{erf}(b\mu)\right]$$

Start with

$$e^{-b^2\mu^2}\int_0^\infty \frac{x^2 e^{-\mu^2 x^2}\,dx}{x^2+b^2} = \int_0^\infty \frac{x^2 e^{-\mu^2(x^2+b^2)}\,dx}{x^2+b^2}.$$

Now observe that

$$\frac{\partial}{\partial\mu}\left(\int_0^\infty \frac{x^2 e^{-\mu^2(x^2+b^2)}\,dx}{x^2+b^2}\right) = -2\mu e^{-\mu^2 b^2}\int_0^\infty x^2 e^{-\mu^2 x^2}\,dx.$$

The change of variables $t=\mu x$ gives

$$\frac{\partial}{\partial\mu}\left(\int_0^\infty \frac{x^2 e^{-\mu^2(x^2+b^2)}\,dx}{x^2+b^2}\right) = -\frac{2e^{-\mu^2 b^2}}{\mu^2}\int_0^\infty t^2 e^{-t^2}\,dt.$$

Integration by parts shows that the last integral is $\sqrt{\pi}/4$, therefore

$$\frac{\partial}{\partial\mu}\left(\int_0^\infty \frac{x^2 e^{-\mu^2(x^2+b^2)}\,dx}{x^2+b^2}\right) = -\frac{\sqrt{\pi}}{2}\frac{e^{-\mu^2 b^2}}{\mu^2}.$$

Now integrate from μ to ∞ and use formula **3.461.5**:

$$\int_\mu^\infty e^{-b^2x^2}\frac{dx}{x^2} = \frac{1}{\mu}e^{-b^2\mu^2} - \sqrt{\pi}b\left[1-\text{erf}(b\mu)\right]$$

to obtain the result.