PROOF OF FORMULA 3.468.2

$$\int_0^\infty \frac{xe^{-\mu x^2}\,dx}{\sqrt{a^2+x^2}} = \frac{1}{2}\sqrt{\frac{\pi}{\mu}}e^{a^2\mu}\left(1-\operatorname{erf}(a\sqrt{\mu})\right)$$

Let $x = a\,\tan t$ to produce

$$\int_0^\infty \frac{xe^{-\mu x^2}\,dx}{\sqrt{a^2+x^2}} = ae^{\mu a^2}\int_0^{\pi/2} \sec t\,\tan t\,e^{-\mu a^2\sec^2 t}\,dt.$$

The change of variables $v = \sec t$ followed by $s = \sqrt{\mu}av$ yield

$$\int_0^\infty \frac{xe^{-\mu x^2}\,dx}{\sqrt{a^2+x^2}} = \frac{e^{\mu a^2}}{\sqrt{\mu}}\int_{a\sqrt{\mu}}^\infty e^{-s^2}\,ds.$$

Now use

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-s^2}\,ds$$

to obtain the result.