PROOF OF FORMULA 3.522.3

$$\int_0^\infty \frac{dx}{(b^2+x^2)\,\cosh ax} = \frac{2\pi}{b}\sum_{k=1}^\infty \frac{(-1)^{k-1}}{2ab+(2k-1)\pi}$$

The change of variables $x = bt$ shows that is suffices to assume $b = 1$. Extend the integrand f to the whole real line using its symmetry. The integrand has poles in the upper half plane at $z = i$ and $z = \frac{(2k+1)\pi i}{2a}$, $k \geq 0$. Assume first that the poles are simple.

Integrate over a semi-circle centered at the origin and radius R. The residues are

$$\begin{aligned}
\text{Res}(f;i) &= \frac{1}{2i\cosh(ia)} = \frac{1}{2i\cos a}, \\
\text{Res}\left(f;\frac{(2k-1)\pi i}{2a}\right) &= \frac{(-1)^{k-1}4ia}{4a^2-\pi^2(2k-1)^2}.
\end{aligned}$$

The residue theorem and a partial fraction decomposition give the stated value of the integral.