PROOF OF FORMULA 3.523.11

$$\int_0^\infty \frac{\sqrt{x}\,dx}{\cosh x} = \sqrt{\pi}\sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^{3/2}}$$

The change of variables $t = \sqrt{x}$ gives

$$\int_0^\infty \frac{\sqrt{x}\,dx}{\cosh x} = 2\int_0^\infty \frac{t^2\,dt}{\cosh(t^2)}$$

that can be written as

$$\begin{aligned} 4\int_0^\infty \frac{t^2 e^{-t^2}\,dt}{1+e^{-2t^2}} &= 4\int_0^\infty t^2 e^{-t^2}\sum_{k=0}^\infty (-1)^k e^{-2kt^2}\,dt \\ &= 4\sum_{k=0}^\infty (-1)^k \int_0^\infty t^2 e^{-(2k+1)t^2}\,dt. \end{aligned}$$

The change of variables $t = u/\sqrt{2k+1}$ and the elementary integral

$$\int_0^\infty u^2 e^{-u^2}\,du = \frac{\sqrt{\pi}}{4},$$

complete the evaluation.