# PROOF OF FORMULA 3.523.12

$$\int_0^\infty \frac{dx}{\sqrt{x}\,\cosh x} = 2\sqrt{\pi}\sum_{k=0}^\infty \frac{(-1)^k}{\sqrt{2k+1}}$$

Let $x = t^2$ to obtain

$$\int_0^\infty \frac{dx}{\sqrt{x}\,\cosh x} = 2\int_0^\infty \frac{dt}{\cosh(t^2)}.$$

The value of this last integral appears in formula **3.511.8**. It is established by expanding the integrand in

$$\int_0^\infty \frac{dt}{\cosh(t^2)} = 2\int_0^\infty \frac{e^{-x^2}\,dx}{1+e^{-2x^2}}$$

as a power series to obtain

$$\int_0^\infty \frac{e^{-x^2}\,dx}{1+e^{-2x^2}} = \sum_{k=0}^\infty (-1)^k \int_0^\infty e^{-(2k+1)x^2}\,dx.$$

The change of variables $x = t/\sqrt{2k+1}$ gives

$$\int_0^\infty \frac{e^{-x^2}\,dx}{1+e^{-2x^2}} = \sum_{k=0}^\infty \frac{(-1)^k}{\sqrt{2k+1}} \int_0^\infty e^{-t^2}\,dt.$$

This last integral has value $\sqrt{\pi}/2$. The formula has been established.