PROOF OF FORMULA 3.527.15

$$\int_0^\infty \frac{\tanh(x/2)}{\cosh x}\, dx = \ln 2$$

Start with

$$\int_0^\infty \frac{\tanh(x/2)}{\cosh x}\, dx = 2\int_0^\infty \frac{e^x - 1}{e^x + 1}\frac{e^x}{e^{2x} + 1}\, dx,$$

and let $t = e^{-x}$ to obtain

$$2\int_0^\infty \frac{e^x - 1}{e^x + 1}\frac{e^x}{e^{2x} + 1}\, dx = 2\int_0^1 \frac{1 - t}{(1 + t)(1 + t^2)}\, dt.$$

Expanding by partial fractions it follows that

$$2\int_0^1 \frac{1 - t}{(1 + t)(1 + t^2)}\, dt = 2\int_0^1 \frac{dt}{1 + t} - 2\int_0^1 \frac{t\, dt}{1 + t^2} = \int_0^1 \frac{dt}{1 + t} = \ln 2.$$