PROOF OF FORMULA 3.541.8

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{\cosh^2 x} = \mu\beta\left(\frac{\mu}{2}\right) - 1$$

Write the integral as

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{\cosh^2 x} = 4\int_0^\infty \frac{e^{-(\mu+2)x}\,dx}{(1+e^{-2x})^2}.$$

Now use

$$\frac{d}{dx}\frac{1}{1+e^{-2x}} = \frac{2e^{-2x}}{(1+e^{-2x})^2}$$

to integrate by parts and obtain

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{\cosh^2 x} = -1 + 2\mu\int_0^\infty \frac{e^{-\mu x}\,dx}{1+e^{-2x}}.$$

The change of variables $t = e^{-2x}$ gives

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{\cosh^2 x} = -1 + \mu\int_0^1 \frac{t^{\mu/2-1}\,dt}{1+t}$$

and this is the result. The β function is

$$\beta(x) = \int_0^1 \frac{s^{x-1}\,ds}{1+s}.$$