PROOF OF FORMULA 3.554.1

$$\int_0^\infty e^{-bx}(1-\text{sech } x)\frac{dx}{x} = 2\ln\Gamma\left(\frac{b+3}{4}\right) - 2\ln\Gamma\left(\frac{b+1}{4}\right) - \ln\frac{b}{4}$$

The integral is expressed as

$$\int_0^\infty e^{-bx}(1-\text{sech } x)\frac{dx}{x} = \int_0^\infty \frac{e^{-bx} - 2e^{-(b+1)x} + e^{-(b+2)x}}{1+e^{-2x}}\,\frac{dx}{x}.$$

The change of variables $t = 2x$ gives

$$\int_0^\infty e^{-bx}(1-\text{sech } x)\frac{dx}{x} = \int_0^\infty \frac{e^{-bt/2} - 2e^{-(b+1)t/2} + e^{-(b+2)t/2}}{1+e^{-t}}\,\frac{dt}{t}.$$

Entry **3.411.28** gives

$$\int_0^\infty \frac{e^{-\nu x} - e^{-\mu x}}{1+e^{-x}}\,\frac{dx}{x} = \ln\left[\frac{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\mu+1}{2}\right)}{\Gamma\left(\frac{\mu}{2}\right)\Gamma\left(\frac{\nu+1}{2}\right)}\right].$$

This yields

$$\int_0^\infty e^{-bx}(1-\text{sech } x)\frac{dx}{x} = \ln\left[\frac{\Gamma\left(\frac{b}{4}\right)\Gamma\left(\frac{b+3}{4}\right)}{\Gamma\left(\frac{b+1}{4}\right)\Gamma\left(\frac{b+2}{4}\right)}\right] + \ln\left[\frac{\Gamma\left(\frac{b+2}{4}\right)\Gamma\left(\frac{b+3}{4}\right)}{\Gamma\left(\frac{b+1}{4}\right)\Gamma\left(\frac{b+4}{4}\right)}\right]$$

that can be reduced to the stated answer.