PROOF OF FORMULA 3.612.2

$$\int_0^{\pi} \frac{\sin nx}{\sin x}\, dx = \begin{cases} 0 & \text{if } n \text{ is even} \\ \pi & \text{if } n \text{ is odd} \end{cases} \qquad n \geq 0$$

Proof. Assume $n \geq 2$ and use

$$\sin nx = \sin(n-1)x \cos x + \cos(n-1)x \sin x$$

to obtain

$$\int_0^{\pi} \frac{\sin nx}{\sin x}\, dx = \int_0^{\pi} \frac{\sin(n-1)x\, \cos x}{\sin x}\, dx + \int_0^{\pi} \cos(n-1)x\, dx.$$

The last integral vanishes. Now use

$$\sin(n-1)x\, \cos x = \frac{1}{2}\left(\sin nx + \sin(n-2)x\right)$$

to obtain

$$\int_0^{\pi} \frac{\sin nx}{\sin x}\, dx = \int_0^{\pi} \frac{\sin(n-2)x}{\sin x}\, dx.$$

The result now follows by induction. □