

PROOF OF FORMULA 3.621.3

$$\int_0^{\pi/2} \sin^{2m} x \, dx = \int_0^{\pi/2} \cos^{2m} x \, dx = \frac{(2m-1)!! \pi}{(2m)!!} \frac{1}{2}$$

Let $t = \sin x$ to obtain

$$\int_0^{\pi/2} \sin^{2m} x \, dx = \int_0^1 t^{2m} (1-t^2)^{-1/2} dt.$$

The change of variables $s = t^2$ gives

$$\int_0^{\pi/2} \sin^{2m} x \, dx = \frac{1}{2} \int_0^1 s^{m-1/2} (1-s)^{-1/2} ds = \frac{1}{2} B\left(m + \frac{1}{2}, \frac{1}{2}\right).$$

Therefore

$$\int_0^{\pi/2} \sin^{2m} x \, dx = \frac{\Gamma(m+1/2)\Gamma(1/2)}{2\Gamma(m+1)}.$$

The result now follows from

$$\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^m} (2m-1)!!$$