PROOF OF FORMULA 3.622.3

$$\int_0^{\pi/4} \tan^{2n} x\, dx = (-1)^n \frac{\pi}{4} + \sum_{k=0}^{n-1} \frac{(-1)^k}{2n-2k-1}$$

Let

$$I_n = \int_0^{\pi/4} \tan^{2n} x\, dx.$$

Then

$$\begin{aligned} I_n &= \int_0^{\pi/4} \tan^{2n} x\, dx = \int_0^{\pi/4} \tan^{2n-2} x \left(\sec^2 x - 1\right)\, dx \\ &= -I_{n-1} + \frac{1}{2n-1}. \end{aligned}$$

Therefore,

$$I_n + I_{n-1} = \frac{1}{2n-1},$$

and $I_0 = \frac{\pi}{4}$. The formula can now be checked directly by induction.

Note. The evaluation should be written as

$$\int_0^{\pi/4} \tan^{2n} x\, dx = (-1)^n \left(\frac{\pi}{4} - \sum_{k=0}^{n-1} \frac{(-1)^k}{2k+1} \right)$$