PROOF OF FORMULA 3.626.2

$$\int_0^{\pi/4} \frac{\sin^{2n} x}{\cos^{2n+3} x} \sqrt{\cos 2x}\, dx = \frac{(2n-1)!!}{(2n+2)!!} \frac{\pi}{2}$$

Let $u = \tan x$ and use $\cos x = 1/\sqrt{1+u^2}$, $\cos 2x = (1-u^2)/(1+u^2)$ and $dx = du/(1+u^2)$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{2n} x}{\cos^{2n+3} x} \sqrt{\cos 2x}\, dx = \int_0^1 u^{2n}(1-u^2)^{1/2}\, du.$$

The change of variable $v = u^2$ gives

$$\int_0^1 u^{2n}(1-u^2)^{1/2}\, du = \frac{1}{2} \int_0^1 v^{n-1/2}(1-v)^{1/2}\, dv = \frac{1}{2} B\left(n + \tfrac{1}{2}, \tfrac{3}{2}\right).$$

The result now follows from

$$B(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$$

and

$$\Gamma\left(n + \tfrac{1}{2}\right) = \frac{(2n-1)!!\,\sqrt{\pi}}{2^n}.$$