PROOF OF FORMULA 3.638.2

$$\int_0^{\pi/4} \frac{\sin^{\mu-1/2} 2x \, dx}{\cos^{\mu} 2x \, \cos x} = \frac{2}{\sqrt{\pi}(2\mu-1)} \Gamma(\mu+\tfrac{1}{2})\Gamma(1-\mu) \sin\left(\frac{(2\mu-1)\pi}{4}\right)$$

The change of variables $t = \tan x$ yields

$$\int_0^{\pi/4} \frac{\sin^{\mu-1/2} 2x \, dx}{\cos^{\mu} 2x \, \cos x} = 2^{\mu-1/2} \int_0^1 t^{\mu-1/2}(1-t^2)^{-\mu} \, dt.$$

Now let $v = t^2$ to obtain

$$\int_0^{\pi/4} \frac{\sin^{\mu-1/2} 2x \, dx}{\cos^{\mu} 2x \, \cos x} = 2^{\mu-3/2} \int_0^1 v^{\mu/2-3/4}(1-v)^{-\mu} \, dv.$$

This last integral is

$$B\left(\tfrac{\mu}{2}+\tfrac{1}{4}, 1-\mu\right) = \frac{\Gamma\left(\frac{\mu}{2}+\frac{1}{4}\right)\Gamma(1-\mu)}{\Gamma\left(\frac{5}{4}-\frac{\mu}{4}\right)}.$$

The result is obtained by using

$$\Gamma(x+\tfrac{1}{2}) = \frac{\Gamma(2x)\sqrt{\pi}}{2^{2x-1}\,\Gamma(x)}$$

to write

$$\Gamma(\tfrac{1}{4}+\tfrac{\mu}{2}) = \frac{\Gamma(\mu-\frac{1}{2})\sqrt{\pi}}{2^{\mu-\frac{1}{2}}\,\Gamma(\frac{\mu}{2}-\frac{1}{4})}.$$