PROOF OF FORMULA 3.644.2

$$\int_0^\pi \frac{\sin^m x\,dx}{1+\cos x} = 2^{m-1}B\left(\frac{m-1}{2}, \frac{m+1}{2}\right)$$

Write the integral as

$$\int_0^\pi \frac{\sin^m x\,dx}{1+\cos x} = \int_0^\pi (1-\cos x)^{m/2}(1+\cos x)^{m/2-1}\,dx.$$

The change of variables $t = 1-\cos x$ produces

$$\begin{aligned}\int_0^\pi \frac{\sin^m x\,dx}{1+\cos x} &= \int_0^\pi (1-\cos x)^{m/2}(1+\cos x)^{m/2-1}\,dx \\ &= \int_0^2 t^{(m-1)/2}(2-t)^{(m-3)/2}\,dt \\ &= 2^{m-1}\int_0^1 s^{(m-1)/2}(1-s)^{(m-3)/2}\,ds\end{aligned}$$

using $t = 2s$ in the last step. This is the result.