

PROOF OF FORMULA 3.662.2

$$\int_0^{\pi/2} (\cosec x - 1)^\mu \sin 2x dx = \frac{(1-\mu)\pi\mu}{\sin \pi\mu}$$

Write

$$\int_0^{\pi/2} (\cosec x - 1)^\mu \sin 2x dx = 2 \int_0^{\pi/2} (1 - \cos x)^\mu \sin x \cos^{1-\mu} x dx.$$

The change of variable $t = \cos x$ gives

$$\int_0^{\pi/2} (\cosec x - 1)^\mu \sin 2x dx = 2 \int_0^1 (1-t)^\mu t^{1-\mu} dt.$$

This last integral is

$$2B(1+\mu, 2-\mu) = \frac{2\Gamma(1+\mu)\Gamma(2-\mu)}{\Gamma(3)},$$

and this simplifies to the stated answer.