PROOF OF FORMULA 3.691.2

$$\int_0^1 \sin(ax^2)\,dx = \sqrt{\frac{\pi}{2a}}S(\sqrt{a})$$

The *Fresnel sine integral* S is defined by

$$S(x) := \sqrt{\frac{2}{\pi}}\int_0^x \sin(t^2)\,dt.$$

The change of variables $t = \sqrt{a}x$ gives

$$\int_0^1 \sin(ax^2)\,dx = \frac{1}{\sqrt{a}}\int_0^{\sqrt{a}} \sin(t^2)\,dt.$$

This yields the result.