FORMULA 4.223.1

$$\int_0^\infty \ln(1 + e^{-x}) \, dx = \frac{\pi^2}{12}$$

Let $t = e^{-x}$ to obtain

$$\int_0^\infty \ln(1 + e^{-x}) \, dx = \int_0^1 \frac{\ln(1+t)}{t} dt.$$

This integral is evaluated in 4.291.1. Its value is $\pi^2/12$.