FORMULA 4.227.4

$$\int_0^{\pi/4} \ln^n \tan x \, dx = (-1)^n n! \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^{n+1}}$$
$$= \frac{1}{2} \left(\frac{\pi}{2}\right)^{n+1} |E_n| \text{ if } n \text{ is even}$$