PROOF OF FORMULA 4.229.1

$$\int_0^1 \ln(\ln 1/x) dx = -\gamma$$

Entry 4.229.4 states that

$$\int_0^1 \ln(\ln 1/x) \left(\ln 1/x\right)^{\mu-1} \, dx = \psi(\mu) \Gamma(\mu)$$
 The result now follows from the values
 $\psi(1) = -\gamma.$