PROOF OF FORMULA 4.231.8

$$\int_0^\infty \frac{\ln x\,dx}{a^2+b^2x^2} = \frac{\pi}{2ab}\ln\frac{a}{b}$$

Formula **4.231.7** states that

$$\int_0^\infty \frac{\ln x\,dx}{(a^2+b^2x^2)^n} = \frac{\Gamma(n-\frac{1}{2})\sqrt{\pi}}{4(n-1)!\,a^{2n-1}b}\left[2\ln\frac{a}{2b} - \gamma - \psi\left(n-\tfrac{1}{2}\right)\right].$$

In the special case $n=1$, this gives

$$\int_0^\infty \frac{\ln x\,dx}{a^2+b^2x^2} = \frac{\pi}{4ab}\left[2\ln\left(\frac{a}{2b}\right) - \gamma - \psi\left(\tfrac{1}{2}\right)\right].$$

The special value

$$\psi\left(\tfrac{1}{2}\right) = -\gamma - 2\ln 2$$

gives the result.