PROOF OF FORMULA 4.244.1

$$\int_0^1 \frac{\ln x\, dx}{\sqrt[3]{x(1-x^2)^2}} = -\frac{1}{8}\left[\Gamma\left(\frac{1}{3}\right)\right]^3$$

In the proof of formula **4.253.1** it was shown that

$$\int_0^1 t^{a-1}(1-t)^{b-1}\ln t\, dt = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}\left[\psi(a)-\psi(a+b)\right].$$

The change of variable $t = x^2$ gives

$$\int_0^1 \frac{\ln x\, dx}{\sqrt[3]{x(1-x^2)^2}} = \frac{1}{4}\int_0^1 t^{-2/3}(1-t)^{-2/3}\,\ln t\, dt.$$

It follows that

$$\int_0^1 \frac{\ln x\, dx}{\sqrt{x(1-x^2)}} = \frac{1}{4}\frac{\Gamma(1/3)\Gamma(1/3)}{\Gamma(2/3)}\left[\psi(1/3)-\psi(2/3)\right].$$

The identities

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x} \text{ and } \psi(x)-\psi(1-x) = -\pi\cot(\pi x),$$

is now used to produce the result.