

PROOF OF FORMULA 4.254.4

$$\int_0^1 \frac{x^{p-1} \ln x}{1+x^q} dx = \frac{1}{q^2} \beta' \left(\frac{p}{q} \right)$$

The change of variable $t = x^q$ gives

$$\int_0^1 \frac{x^{p-1} \ln x}{1+x^q} dx = \frac{1}{q^2} \int_0^1 \frac{t^{p/q-1} \ln t}{1+t} dt.$$

The beta function is defined by

$$\beta(a) = \int_0^1 \frac{t^{a-1} dt}{1+t}.$$

Therefore

$$\beta'(a) = \int_0^1 \frac{t^{a-1} \ln t}{1+t} dt.$$

The result follows from this expression.