PROOF OF FORMULA 4.257.2

$$\int_0^\infty \left(\frac{x^p}{q^{2p}+x^{2p}}\right) \ln\frac{x}{q}\,\frac{dx}{x} = 0$$

Let $x = qt$ to obtain

$$\int_0^\infty \left(\frac{x^p}{q^{2p}+x^{2p}}\right) \ln\frac{x}{q}\,\frac{dx}{x} = \frac{1}{q^p}\int_0^\infty \frac{t^p}{1+t^{2p}}\,\frac{\ln t}{t}\,dt$$

Split the integral into $[0,1]$ and $[1,\infty)$. In the second interval let $s = 1/t$ and check that its value is minus the integral over $[0,1]$. Thus the total integral vanishes.