PROOF OF FORMULA 4.267.2

$$\int_0^1 \frac{(1-x)^2}{1+x^2} \, \frac{dx}{\ln x} = \ln \frac{\pi}{4}$$

The change of variables $t = -lnx$ gives

$$\int_0^1 \frac{(1-x)^2}{1+x^2} \, \frac{dx}{\ln x} = -\int_0^\infty \frac{(1-e^{-t})^2 e^{-t}}{t(1+e^{-2t})} \, dt.$$

Now let $s = 2t$ to obtain

$$\int_0^1 \frac{(1-x)^2}{1+x^2} \, \frac{dx}{\ln x} = -\int_0^\infty \frac{e^{-s/2} - e^{-s}}{1+e^{-s}} \, \frac{ds}{s} + \int_0^\infty \frac{e^{-s} - e^{-3s/2}}{1+e^{-s}} \, \frac{ds}{s}.$$

Entry **3.411.28** shows that this integral has the value

$$-\ln \left(\frac{\Gamma(1/4)\Gamma(1)}{\Gamma(1/2)\Gamma(3/4)} \right) + \ln \left(\frac{\Gamma(1/2)\Gamma(5/4)}{\Gamma(3/4)\Gamma(1)} \right) = \ln \frac{\pi}{4}.$$

This proves the result.