PROOF OF FORMULA 4.269.7

$$\int_u^v \frac{dx}{x\sqrt{\ln(x/u)\,\ln(v/x)}} = \pi$$

The change of variables $t = x/u$ gives
$$\int_u^v \frac{dx}{x\sqrt{\ln(x/u)\,\ln(v/x)}} = \int_1^c \frac{dt}{t\sqrt{\ln t\,(\ln c - \ln t)}}.$$
where $c = v/u$. Now let $z = \ln t$ to obtain
$$\int_1^c \frac{dt}{t\sqrt{\ln t\,(\ln c - \ln t)}} = \int_0^a \frac{dz}{z(a-z)},$$
with $a = \ln c$. The change of variables $z = a\sin^2\varphi$ gives the result.