

PROOF OF FORMULA 4.271.3

$$\int_0^1 \frac{\ln^{2n-1} x \, dx}{1-x} = -\frac{2^{2n-2}}{n} \pi^{2n} |B_{2n}|$$

Entry **4.271.4** states that

$$\int_0^1 \frac{\ln^{p-1} x \, dx}{1-x} = (-1)^{p-1} \Gamma(p) \zeta(p).$$

The special value $p = 2n$ gives the result, using the relation

$$\zeta(2n) = \frac{2^{2n-1} \pi^{2n} |B_{2n}|}{(2n)!}.$$